## A Demonstration of Wave Fronts, Attenuation, and Diffraction*

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The concept of wave fronts, defined as points of like phase, can be demonstrated by the use of an audiofrequency oscillator, a loud-speaker, a microphone, and a cathode-ray oscilloscope. The shapes of wave fronts radiated from plane, line, and point sources and the attenuation of intensity with distance can be qualitatively demonstrated. The double slit diffraction pattern can be explored as a function of the wavelength, slit spacing, and distance to the plane of observation in a simple arrangement of equipment.

A N audiofrequency oscillator, a large diameter loud-speaker with appropriate orifice masks, a microphone, and a cathode-ray oscilloscope can be used to obtain a very simple demonstration of most of the relationships associated with the propagation of sinusoidal waves. Multiple-exposure photographs provide a convenient form for recording the results of the experiment.

The equipment is connected as shown in Fig. 1. If the distance $d$ is such that the sound pressure
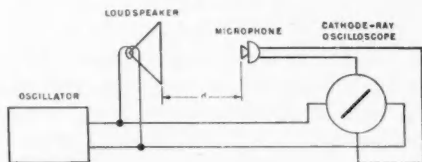


FIG. 1. Relative phase display: The output of the audiofrequency oscillator provides at the oscilloscope a reference phase against which the phase of the microphone output is compared. The microphone-loud-speaker distance, $d$, is varied to demonstrate the phase relationship between the loud-speaker and the microphone and its effect on the oscilloscope pattern.

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at the microphone is a maximum at the same time that the voltage across the driving coil of the loud-speaker is a maximum, the spot on the oscilloscope will trace a straight line as the
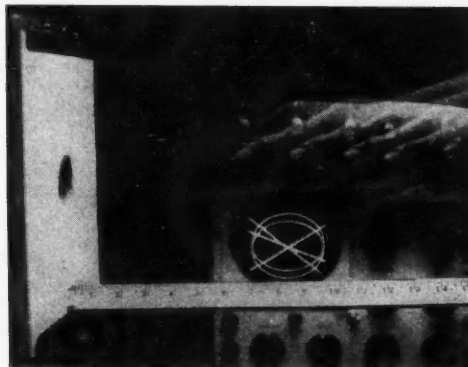


FIG. 2. Relative phase at points along a wavelength: Beginning with the straight line of maximum negative slope, successive displacements of the microphone away from the sound source by one-quarter wavelength intervals yield corresponding patterns of a circle, a straight line with positive slope, an ellipse with the major axis horizontal, and a straight line having reduced negative slope as the full wavelength is completed.
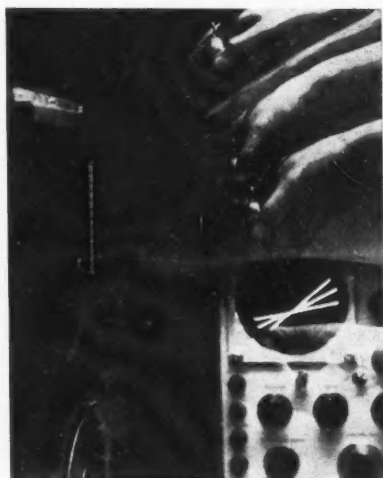
FIG. 3. Wave front radiated by a large diaphragm: Points of like phase defining a wave front are located by varying the spacing of the microphone from the loud-speaker in such a manner as to keep the relative phase display on the oscilloscope a straight line. The slope of the line will vary with the variations in intensity of the sound wave at different points in the wave front.

magnitude of the pressure and voltage vary throughout a cycle of the audiofrequency being radiated by the loud-speaker. After establishing the principle of the oscilloscope display, the first step in the demonstration consists of determining the distance between points of like phase along the path of the radiating wave. With the maximum vertical and horizontal displacements
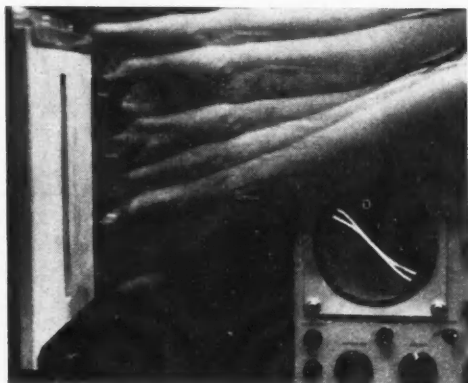


FIG. 4. Wave front in a plane parallel to a line source: The wave front in a plane containing the slit and the loud-speaker axis is found to be similar to the wave front from the unmasked loud-speaker except with respect to intensity. Increased audio power or microphone gain must be used.

on the oscilloscope set about equal, the most convenient reference phases will be those in which the ellipse reduces to a single straight line corresponding to multiples of one-half wavelength between the effective source and the microphone. Figure 2 shows the superimposed wave forms obtained as the sound field is explored over one wavelength. If the wavelength in air of the sound wave has previously been calculated for the test frequency, it is now convenient to give meaning to the concept of 360° per wavelength by watching the pattern on the oscilloscope "rotate" as the spacing of the microphone is varied over several wavelengths.

Having established a concept of *points of like phase*, we are now ready to explore the *wave*
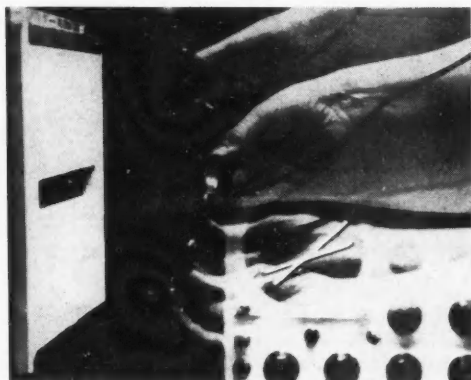


FIG. 5. Wave front in a plane perpendicular to a line source: Cross sections taken at different positions along the slit all show essentially circular wave fronts in planes taken perpendicular to the axis of the slit.

*fronts* radiating from various configurations of sources. Using the unmasked loud-speaker for a source as shown in Fig. 3, the contour of the wave fronts in front of the diaphragm can be determined by moving the microphone over the loud-speaker aperture at a spacing such as to keep the relative phase constant as indicated on the oscilloscope. The process should be repeated at several spacings between the microphone and the loud-speaker and at several frequencies in order to obtain some idea of how far out on the axis the wave front remains reasonably flat.

By using the Celotex aperture masks in the guides in front of the loud-speaker, wave fronts can be explored for a variety of aperture configurations. Figures 3–6, on which lines can be

drawn through points of like phase on the multiple exposure photographs, show the shapes of typical wave fronts radiating from plane, line, and point sources. In all wave front exploration, it will be found best to use the straight line oscilloscope display as a reference phase because any slight deviation from the wave front is immediately apparent.

For the next part of the demonstration, the horizontal sweep amplifier of the oscilloscope is disconnected from the input terminals and connected to the internal linear sweep generator, which is then adjusted to display several cycles of the sound frequency being picked up by the



FIG. 7. Attenuation as a function of distance from a point source: A decrease in the signal by a factor of four results from an increase in the microphone-loud-speaker separation by a factor of two. For good results the waves must radiate freely in all directions.
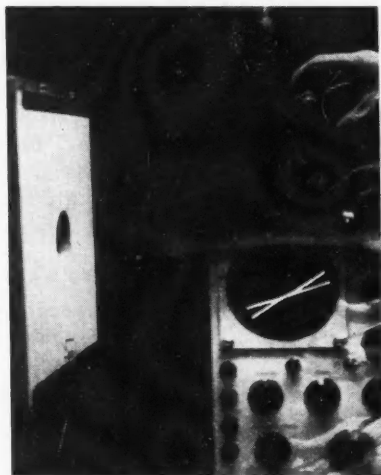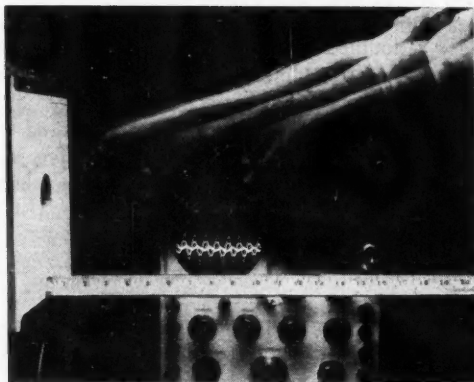


FIG. 6. Wave front radiated by a small circular aperture representing a point source: If the wave front is explored in all planes, the meaning of the expression *spherical wave* will become evident.

microphone. The microphone is moved along the axis of the loud-speaker, and the variations in amplitude of the oscilloscope pattern are taken as a measure of the variations in intensity of the sound wave as a function of distance from the source. If a large diameter loud-speaker or a phased array of small speakers is used, it will be found that the response out of the microphone will be reasonably constant over a short distance, $d$, which corresponds roughly to the distance over which the plane wave-front had previously been found to exist. This demonstration provides qualitative verification that sound intensity, like other forms of radiant energy intensity, is *inde-*

*pendent* of the distance from a *plane source* having dimensions which are large compared to the test distance.

The same procedure is now repeated using the line source, and qualitative confirmation will be found for the relationship that the intensity of a radiation attenuates as the *inverse first power* of the distance from a *line source* whose length is large compared to the test distance. Repeating the experiment using the *point source* yields the familiar *inverse square law*. In Fig. 7 the intervals
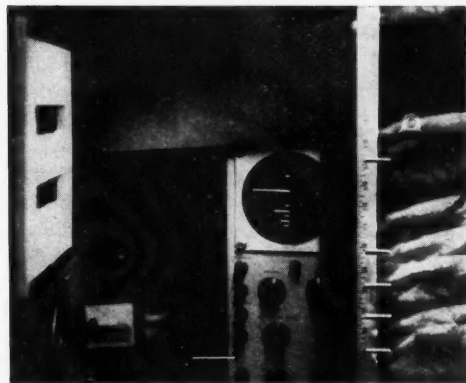


FIG. 8. Double slit diffraction pattern: Alternate cancelation and reinforcement of the sound waves occur as the microphone is moved vertically so as to change the difference between the distances to the two slits by one-half wavelength. Intensity is measured horizontally on the oscilloscope, and each trace represents one of the positions of the microphone.

of distance have been selected so that the distance is doubled for each step. The sound waves must be free to radiate in all directions in order to produce consistent results.

The double slit shown in Fig. 8 can be used to demonstrate reinforcement and destructive interference and will produce a diffraction pattern. An audiofrequency of about 6000 cps should be used so that the condition for one-half wavelength difference in path length between the waves from the two slits will not require too large an angular displacement from the axis. Exploration with the microphone, preferably one with a small opening to assure good nulls, is made in a plane containing the loud-speaker axis and oriented perpendicular to the slits. To obtain the photograph in Fig. 8 the microphone was connected to the horizontal axis of the oscilloscope through an auxiliary amplifier so that

intensities corresponding to positions of the microphone could be displayed in the same relative positions as the displacements of the microphone. The horizontal centering was shifted for each trace so as to align the right-hand edges of the traces. The derivation of the equations giving the envelope of the pattern shown is to be found in *Fundamentals of Physical Optics*, by Jenkins and White.[1]

The phenomena demonstrated here apply equally well to sound, light, or radio wave propagation. Experiments of this type give the physics student a dynamic visual experience to which he will return again and again for understanding as the associated mathematical theory is developed in his further undergraduate and graduate studies.

[1] F. A. Jenkins and H. E. White, *Fundamentals of Physical Optics* (McGraw-Hill Book Company, Inc., New York, 1937), first edition, pp. 131 and 133.

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## Comic Dictionary of Physical Terms

Illustrated by MARGE M. MULLER
*East Lansing, Michigan*



Constant Deviation

Discrete State
$M^3$

Degenerate State

Reno

Las Vegas

$M^3$

Excited State
$M^3$

# On a Degeneracy Theorem of Kramers

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There is a theorem due to Kramers which states that the energy states of systems with an odd number of electrons remain at least doubly degenerate in the presence of any purely electric fields. The physical significance of this theorem, its proof, and a discussion of the relationship of the Kramers degeneracy and the Wigner time-reversal operation are considered. A proof is given of the theorem that the expectation value of the magnetic moment operator is zero for any nondegenerate state.

ONE of the important problems in the adiabatic demagnetization method of attaining very low temperatures (well below 1°K) is the choice of paramagnetic salts to be used.[1] The salts are chosen primarily for their magnetic susceptibilities and specific heats, which properties are determined by the structure of the energy spectrum (at these temperatures, only the lowest energy levels) of the paramagnetic ions in the salt crystal. The energy states of the ion in the crystal are in general different from those of the free ion, the latter being split by the influence of the strong electric fields within the crystal (crystalline Stark effect). Now there is a theorem due to Kramers[2] which enables one to state that in certain cases the crystalline electric fields are ineffective in splitting the degenerate states of the free ions; Kramers' theorem tells us that the energy states of ions with an odd number of electrons are at least doubly degenerate even in the presence of any electric fields.

In view of what has been said it follows that Kramers' theorem is of considerable importance in the study of the adiabatic demagnetization method and, therefore, in low temperature physics generally. In addition, it has been shown by Wigner[3] that the Kramers degeneracy is connected to an important symmetry property of quantal systems in purely electric fields— invariance under the operation of time reversal.

In spite of the intrinsic interest and important applications of Kramers' theorem, it is not discussed in detail in the literature[4] except for the original papers of Kramers and Wigner. It is the purpose of the present paper to provide a discussion of the theorem and some of its consequences which may be of interest to physicists in general and of direct use to students and workers in the quantum theory of paramagnetism and low temperature physics.

With this purpose in mind we have given first a physical discussion of Kramers' theorem and its connection with time reversal invariance. This is followed by a detailed mathematical discussion in which proofs are given of the theorem and its important corollary on the magnetic properties of nondegenerate states.

## PHYSICAL DISCUSSION

The existence of degeneracy in the energy states of a quantal system, when this degeneracy is not an accidental consequence of the numerical values of the parameters of the system, is closely related to the invariance or symmetry properties of the system.[5] If the Hamiltonian of the system is invariant under some operation, then performing this operation on an eigenfunction of the Hamiltonian will give rise to a new function which is also an eigenfunction of the Hamiltonian corresponding to the same energy eigenvalue. This may be seen directly from the follow-

[1] For general discussion see H. B. G. Casimir, *Magnetism and Very Low Temperatures* (Cambridge University Press, Cambridge, 1940). Also Burton, Grayson, Smith, and Wilhelm, *Phenomena at the Temperature of Liquid Helium* (Reinhold Publishing Corporation, New York, 1940).

[2] H. A. Kramers, Proc. Amsterdam Acad. **33**, 959 (1930).

[3] E. Wigner, Nachr. Akad. Wiss. Göttingen, Math.-physik. Kl. (1932), 546.

[4] The theorem is given (Reference 1) without proof by Casimir, p. 53, and by Burton *et al.*, p. 142 and p. 182. See also J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Oxford University Press, Oxford, 1932), p. 296, and H. A. Kramers, *Hand und Jahrbuch der Chemischen Physik* (Akademische Verlagsgesellschaft, Leipzig, 1938), Band 1, p. 390.

[5] See, for example, E. C. Kemble, *The Fundamental Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), p. 310.

ing argument. Let $H$ be the Hamiltonian, $A$ the operator in question, and $\psi$ an eigenfunction of $H$ corresponding to energy $E$. Then,

$$H\psi = E\psi. \tag{1}$$

Operating with $A$ on both sides gives us

$$AH\psi = (AHA^{-1})A\psi = E(A\psi), \tag{2}$$

where we have inserted the factor $A^{-1}A$, equal to the unit operator, between $H$ and $\psi$. Now if $H$ is invariant under $A$, that is, if

$$AHA^{-1} = H, \quad \text{and} \quad AH = HA, \tag{3}$$

it follows from Eq. (2) that $A\psi$ is an eigenfunction of $H$ corresponding to energy $E$. (We may note that even if Eq. (3) is not satisfied, $A$ will be an eigenfunction of $AHA^{-1}$ corresponding to the energy eigenvalue $E$.)

This argument does not prove that the invariance of the Hamiltonian under the operation $A$ necessarily implies degeneracy of the energy levels, because it is possible that $A\psi$ is a function not essentially different from $\psi$, that is, $A\psi$ may be a constant multiple of $\psi$. Only if $\psi$ and $A\psi$ are linearly independent does it follow that $E$ is degenerate. For example, the Hamiltonian of a harmonic oscillator is invariant under the operation which replaces $x$ by $-x$, where $x$ is the oscillator coordinate. Nevertheless the energy states $E_n$ of the harmonic oscillator are not degenerate because the corresponding eigenfunctions $\psi_n$ are either odd or even, so that reversal of the sign of $x$ merely multiplies the wave function $\psi_n$ by the constant $(-1)^n$.

Kramers' theorem, which says that the energy states of a system with an odd number of electrons are at least doubly degenerate in any electric fields, is an assertion of the existence of degeneracy under rather general conditions and one can expect, therefore, a correlated symmetry property. Wigner has shown that this corresponding symmetry property is invariance under time reversal, and we shall now discuss the nature of this invariance.

Let us begin by considering a classical system whose Hamiltonian is $H(q, p)$, where $q$ and $p$, respectively, stand for the generalized coordinates and momenta of the system. It may easily be verified that if $q(t)$, $p(t)$ are a solution of the Hamiltonian equations of motion, then $q(-t)$,

$-p(-t)$ are a solution of the Hamiltonian equations for the modified Hamiltonian $H(q, -p)$. It is clear that $q(-t)$, $-p(-t)$ constitute the time-reversed solution to $q(t)$, $p(t)$, that is, the solution in which the particles carry out the same motion reversed in time. Furthermore, the operation on the Hamiltonian, which produces the time-reversed solution and which may appropriately be called the time-reversal operator, is the operation of reversing the directions of all momenta.

In the absence of velocity-dependent interactions and, in particular, in the absence of magnetic fields, $H(q, -p)$ is equal to $H(q, p)$: the Hamiltonian can be said to be invariant under the operation corresponding to time reversal. In this case the time reversed functions $q(-t)$, $-p(-t)$ are a solution of the original mechanical problem. To sum up, when the Hamiltonian is invariant under the operation discussed, the time-reversed functions constructed from any solution of the Hamiltonian equations are themselves a solution of these same equations of motion.[6]

Now on the basis of the formal similarity between classical and quantum mechanics, which is seen most readily in the Poisson bracket-commutator correspondence, it may be concluded that, in the quantal case as well as the classical case, the operation on the Hamiltonian corresponding to time reversal is the reversal of the directions of all momenta.[6] We may then reasonably expect that when the Hamiltonian is invariant under this operation, there is the possibility of a twofold degeneracy in the solutions of the Schrödinger equation. In particular, we shall prove that if $\psi$ is a solution of the Schrödinger equation, Eq. (1) (where the Hamiltonian is $H(\mathbf{r}, \mathbf{p}, \mathbf{s})$) corresponding to energy $E$, then there is a function $\psi'$ which is a solution of the equation

$$H'\psi' = E\psi' \tag{4}$$

with the same energy $E$. Here $H'$ is $H(\mathbf{r}, -\mathbf{p}, -\mathbf{s})$, with $\mathbf{r}, \mathbf{p}, \mathbf{s}$, respectively, standing for the position vectors, linear momenta, and spin angular

[6] See the discussion in R. C. Tolman, *The Principles of Statistical Mechanics* (Oxford University Press, Oxford, 1938), p. 102 and p. 395. Also see L. Onsager, Phys. Rev. **37**, 405 (1931) and **38**, 2265 (1931) for a discussion of the case when magnetic fields are present.

momenta of all particles. If $H'$ and $H$ are equal, that is, if $H$ is invariant under the time-reversal operation, then $\psi'$ and $\psi$ will both be solutions of Eq. (1) corresponding to the same energy $E$.

It follows from the discussion of Eqs. (1)–(3) that to prove Eq. (4) we must exhibit an operator $K$ which has the property

$$KH(\mathbf{r}, \mathbf{p}, \mathbf{s})K^{-1} = H(\mathbf{r}, -\mathbf{p}, -\mathbf{s}) = H'. \quad (5)$$

For then by operating on Eq. (1) with $K$ we would obtain

$$KH\psi = (KHK^{-1})K\psi = H'(K\psi) = E(K\psi), \quad (6)$$

and the desired function $\psi'$ will simply be

$$\psi' = K\psi. \quad (7)$$

In the following section we shall prove that the operator $K$ exists and we shall give explicit expressions for $K$ and for $\psi'$. The operator $K$ which reverses the directions of all momenta (linear and spin) may appropriately be called the time-reversal operator.

Now, if we have an atomic system subject to purely electric fields, the Hamiltonian will be invariant under the time-reversal operation. For in this case we can choose the vector potential of the external fields to be identically zero. The scalar potential, of course, depends only on the coordinates. The spins and momenta will then appear in only the following two types of terms. First, there are kinetic energy terms of the form $p^2/2m$ and second, there are spin-orbit interaction terms of the form $\mathbf{L} \cdot \mathbf{s} = (\mathbf{r} \times \mathbf{p}) \cdot \mathbf{s}$. It is clear that both types are left unchanged when $\mathbf{p} \rightarrow -\mathbf{p}$ and $\mathbf{s} \rightarrow -\mathbf{s}$. We see then that, in purely electric fields, the Hamiltonian is invariant under the time-reversal operator $K$.

The one essential point about Kramers' theorem which remains to be discussed is the difference in the properties of the operator $K$ for systems containing an odd rather than an even number of electrons. It will be recalled that invariance of the Hamiltonian under the operator $K$ implies degeneracy only if $K\psi$ and $\psi$ are linearly independent. We shall prove in the next section that $K\psi$ and $\psi$ are necessarily linearly independent when we are dealing with an odd electron system whereas no such restriction exists for an even electron system. This statement, in conjunction with the discussion given

above, will suffice to prove Kramers' theorem: the energy states of an odd electron system are necessarily degenerate regardless of the electric fields applied.

Another important theorem[7] closely related to Kramers' theorem is the following: the expectation value of the magnetic moment is zero in any nondegenerate state. This theorem will be proved in the final section of the present paper. The main idea in the proof is, that if a state is nondegenerate, $\psi$ and $K\psi$ are essentially the same function. Hence we must be dealing only with even electron systems.

The theorem just mentioned has significant consequences in the theory of paramagnetic susceptibilities. In general, the paramagnetic susceptibility of a system as given by the Langevin-Debye formula[8] is the sum of two terms. The first term is proportional to the sum of the squares of the diagonal matrix elements (expectation values in pure states) of the magnetic moment operator and varies with temperature as $T^{-1}$. The second term is essentially independent of the temperature and depends on the nondiagonal matrix elements of the magnetic moment operator. Now, if we deal with an ion having an even number of electrons, whose ground state is nondegenerate and to which the above theorem applies, the paramagnetic susceptibility will approach at very low temperatures a constant equal to the second term mentioned above. For an ion with an odd number of electrons, however, the diagonal matrix elements of the magnetic moment operator are not zero and the paramagnetic susceptibility is strongly temperature dependent even as the temperature goes to zero. These predictions have been checked in detail by theoretical calculations of Penney and Schlapp.[9] These authors calculated the susceptibilities of praseodymium and neodymium sulfates and their results are in good agreement with experiment. (The ion $Pr^{+++}$, $Z = 59$ has an even number of electrons; the ion $Nd^{+++}$, $Z = 60$ has an odd number of electrons so the comparison can appropriately be made.)

[7] See reference 2 or reference 4, p. 273.

[8] See Van Vleck, reference 4, Chapters 2, 4, and especially Chapters 7, 9 where appropriate corrections to the L.-D. formula are discussed.

[9] W. G. Penney and R. Schlapp, Phys. Rev. 41, 194 (1932).

Before proceeding to our mathematical discussion there is one other point to be mentioned. For odd-electron systems Kramers' theorem tells us that there is a twofold degeneracy in the energy states which cannot be removed by the otherwise effective crystalline electric fields. Now at temperatures approaching absolute zero this degeneracy must be split by some mechanism. This result follows from Nernst's theorem.[10] For, an external magnetic field will split the degeneracy and, if it were not otherwise split, one could have a nonvanishing change in entropy in the process of magnetization at absolute zero, contradicting Nernst's theorem. There are, however, forces which will split the degeneracy, namely the dipole-dipole forces and the exchange forces between the paramagnetic ions. A considerable amount of work has been done on how these forces act to split the Kramers degeneracy, but the problem is not yet satisfactorily solved.[11]

### MATHEMATICAL DISCUSSION

Before giving the proof of Kramers' theorem we shall enumerate some properties of the electron spin which are used below.[12]

For a one-electron system the general wave function is a two-component quantity which we shall write as

$$\psi(\mathbf{r}, s) = \varphi_{\frac{1}{2}}(\mathbf{r})\xi^{(\frac{1}{2})} + \varphi_{-\frac{1}{2}}(\mathbf{r})\xi^{(-\frac{1}{2})} = \sum_{s=\pm\frac{1}{2}} \varphi_s \xi^{(s)}. \quad (8)$$

Here $s$ is the value, in units of $\hbar = h/2\pi$, of the component of the spin angular momentum in a particular direction (the $z$ direction), which takes on the values $\pm\frac{1}{2}$. On the right-hand side of Eq. (8), $\varphi_s(\mathbf{r})$ is a function of the position coordinates of the electron and $\xi^{(s)}$ is a unit vector in the two-dimensional spin space. We note that

$$\xi^{(s)} \cdot \xi^{(s')} = \delta_{ss'}. \quad (9)$$

The components of the spin angular momentum are the operators $s_x$, $s_y$, $s_z$ which when

expressed in units of $\hbar$, satisfy the equations

$$s_x^2 = s_y^2 = s_z^2 = 1, \quad (10)$$

where 1 is the unit matrix, and

$$s_x s_y = -s_y s_x = i s_z, \quad (11)$$

together with the two equations obtained from Eq. (11) by cyclic permutation of $x$, $y$, $z$. The usual matrices representing $s_x$, $s_y$, $s_z$ are

$$s_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad s_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad s_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (12)$$

Since any matrix of order two contains four elements, the most general matrix is a linear combination of 1, $s_x$, $s_y$, and $s_z$. Thus, the Hamiltonian of a one-electron system can be written in the form

$$H(\mathbf{r}, \mathbf{p}, \mathbf{s}) = H_0(\mathbf{r}, \mathbf{p})1 + H_x(\mathbf{r}, \mathbf{p})s_x \\ + H_y(\mathbf{r}, \mathbf{p})s_y + H_z(\mathbf{r}, \mathbf{p})s_z, \quad (13)$$

where $H_0$, $H_x$, etc., are operators which act on only the spatial coordinates, and $\mathbf{p}$ is the linear momentum operator.

For an $n$-electron system the general wave function has $2^n$ components and may be written as

$$\psi(\mathbf{r}_1, \cdots \mathbf{r}_n, s_1, \cdots s_n) \\ = \sum_{\text{all } s_i = \pm\frac{1}{2}} \varphi_{s_1 \cdots s_n}(\mathbf{r}_1, \cdots \mathbf{r}_n)\xi_1^{(s_1)} \cdots \xi_n^{(s_n)}, \quad (14)$$

where $\varphi_{s_1 \cdots s_n}$ is a function of positional coordinates only and where the summation is over the possible values $(\pm\frac{1}{2})$ of all $s_i$.

Since quadratic and higher powers and also products of the spin matrices can be expressed in terms of the matrices themselves by means of Eqs. (10) and (11), we can write the Hamiltonian operator as

$$H(\mathbf{r}_1, \cdots \mathbf{r}_n, \mathbf{p}_1, \cdots \mathbf{p}_n, \mathbf{s}_1, \cdots \mathbf{s}_n) \\ = H^{(0)} + H^{(1)} + H^{(2)} + \cdots, \quad (15)$$

where

$$H^{(0)} = H^{(0)}(\mathbf{r}, \mathbf{p})1_1, \cdots 1_n,$$

$$H^{(1)} = \sum_{i=1}^{n} (H_{xi}s_{xi} + H_{yi}s_{yi} + H_{zi}s_{zi})\prod_{j \neq i} 1_j, \quad (16)$$

$$H^{(2)} = \sum_{i \neq j} (H_{xixj}s_{xi}s_{xj} + H_{xiyj}s_{xi}s_{yj} + \cdots) \prod_{k \neq i, j} 1_k,$$

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[10] See R. H. Fowler and E. A. Guggenheim, *Statistical Thermodynamics* (Cambridge University Press, Cambridge, 1939), Chapters 5, 14.

[11] See J. M. Luttinger and L. Tisza, Phys. Rev. **70**, 954 (1946) for a recent discussion with references to previous work on the problem.

[12] See, for example, Kemble (reference 5), p. 510 or Kramers (reference 4), Chapter 6. We use the notation of Kramers.

where $1_i$, $s_{xi}$, etc., are matrices acting on the $i$th particle. It will be noticed that every term in the Hamiltonian is the product of $(n+1)$ factors, the first being a spatial operator and the other $n$ being spin matrices (including the unit matrix), one operator for each of the $n$ electrons.

Having given the form of the wave functions and Hamiltonian for the general case, we may now proceed to our theorem.

*Theorem:* There is an operator $K$ such that if $\psi(\mathbf{r}, s)$ as given by Eq. (14), is an eigenfunction of the Hamiltonian $H(\mathbf{r}, \mathbf{p}, \mathbf{s})$ corresponding to energy eigenvalue $E$ then $\psi' = K\psi$ is an eigenfunction of $H(\mathbf{r}, -\mathbf{p}, -\mathbf{s})$ corresponding to the same eigenvalue $E$. (Note that $\mathbf{r}$ stands for the set of vectors $\mathbf{r}_1, \cdots, \mathbf{r}_n$ and similarly for $\mathbf{p}$ and $\mathbf{s}$.)

To prove this, we need only construct an operator $K$ which satisfies Eq. (5) as discussed in the previous section.

Consider first the one-electron case. The operator $K$ can then be written as

$$K = is_y C, \qquad (17)$$

where $C$ is defined by

$$C\varphi(\mathbf{r}) = \varphi^*(\mathbf{r}); \qquad (18)$$

$C$ is the operator which replaces a function by its complex conjugate. Note that $C$ commutes with $is_y$ since from Eq. (12) the latter is real.

To prove that $is_y C$ has the desired property we notice that

$$CHC^{-1} = H^*, \qquad (19)$$

since $C^{-1}$ is equal to $C$, and $(CHC^{-1})\varphi = (CH)\varphi^* = (H^*)\varphi$.

Now $i$ enters the general Hamiltonian of Eq. (13) in $\mathbf{p} = (\hbar/i)\nabla$ and in $s_y$. Hence

$$CHC^{-1} = H_0(\mathbf{r}, -\mathbf{p})1 + H_x(\mathbf{r}, -\mathbf{p})s_x \\ - H_y(\mathbf{r}, -\mathbf{p})s_y + H_z(\mathbf{r}, -\mathbf{p})s_z. \quad (20)$$

Operating on Eq. (20) with $is_y$ we obtain by direct calculation, using Eqs. (10) and (11),

$$(is_y)CHC^{-1}(is_y)^{-1} \\ = H_0(\mathbf{r}, -\mathbf{p})1 - H_x(\mathbf{r}, -\mathbf{p})s_x \\ - H_y(\mathbf{r}, -\mathbf{p})s_y - H_z(\mathbf{r}, -\mathbf{p})s_z \quad (21) \\ = H(\mathbf{r}, -\mathbf{p}, -\mathbf{s}) = H',$$

so that $K = is_y C$ satisfies the required condition.

For the $n$-electron system the operator $K$ is

$$K = i^n s_{y1} \cdots s_{yn} C. \qquad (22)$$

This is now readily verified on the Hamiltonian of Eq. (15); operating with $C$ reverses the signs of all $\mathbf{p}_i$ and all $s_{yi}$ as in Eq. (20). Operating with each factor $is_{yi}$ reverses the signs of the corresponding $s_{xj}$ and $s_{zj}$ as in Eq. (21), since all matrices appear linearly in the Hamiltonian. Hence, this $K$ satisfies Eq. (5) in the $n$-electron case. This construction of $K$ proves our theorem.

The explicit form of $\psi'$ is of some interest.

$$\psi' = i^n s_{y1} \cdots s_{yn} C \sum_{\text{all } s_i = \pm\frac{1}{2}} \varphi_{s_1 \cdots s_n} \xi_1^{(s_1)} \cdots \xi_n^{(s_n)}$$
$$= i^{n-1} s_{y1} \cdots s_{yn-1} \sum (-1)^{\frac{1}{2} - s_n} \varphi^*_{s_1 \cdots s_{n-1} - s_n} \\ \xi_1^{(s_1)} \cdots \xi_n^{(s_n)}$$
$$= \sum (-1)^S \varphi^*_{-s_1 \cdots -s_n} \xi_1^{(s_1)} \cdots \xi_n^{(s_n)}, \qquad (23)$$

where

$$S = n/2 - \sum_{k=1}^{n} s_k.$$

The matrix elements of $is_y$ have been used in this calculation.

To complete the proof of Kramers' theorem we must show that $\psi'$ and $\psi$ are linearly independent for odd electron systems. To do this we first calculate $K^2$.

$$K^2 = (i^n s_{y1} \cdots s_{yn} C)^2 = i^{2n} 1 = (-1)^n 1, \quad (24)$$

where 1 is the unit operator in the $2^n$ dimensional spin space. For even electron systems $K^2$ is the unit operator, for odd electron systems it is the negative of the unit operator. Now assume $\psi'$ and $\psi$ are linearly dependent:

$$\psi' = K\psi = a\psi, \qquad (25)$$

where $a$ is some complex number. Then

$$K^2\psi = K\psi' = Ka\psi = a^*K\psi = |a|^2\psi, \qquad (26)$$

using the fact that $K$ contains the conjugation operator $C$ as a factor. Hence we must have

$$|a|^2 = (-1)^n. \qquad (27)$$

This can be true only if $n$ is even. Hence for odd $n$, $K\psi$ and $\psi$ are necessarily linearly independent and the energy states are degenerate.

## MAGNETIC MOMENTS AND DEGENERACY

This section is devoted to a proof of the following theorem: The expectation value of the magnetic moment operator of an $n$-electron system vanishes for all nondegenerate states.

From Kramers' theorem it follows that $n$ must be even for nondegenerate states to be possible. From the proof of that theorem we can see that, if a state $\psi$ is nondegenerate, then $\psi$ and $\psi'$ are linearly dependent and there is no loss in generality in taking them as equal. Hence from Eq. (23), for a nondegenerate state,

$$\psi = \sum \varphi_{s_1 \cdots s_n} \xi_1^{(s_1)} \cdots \xi_n^{(s_n)} = \psi'$$

$$= \sum (-1)^S \varphi^*_{-s_1 \cdots -s_n} \xi_1^{(s_1)} \cdots \xi_n^{(s_n)}. \quad (28)$$

Our problem is then to prove that

$$(\psi^*, \mathbf{M}\psi) = 0, \quad (29)$$

if Eq. (28) is satisfied, where $\mathbf{M}$ is the magnetic moment operator and the symbol $(\psi_1, \psi_2)$ means the integral over position coordinates (of all electrons) of the scalar product of the two spinor wave functions $\psi_1$ and $\psi_2$.

The magnetic moment operator is the sum of two contributions, one from the orbital angular momenta and one from the spins. It will be shown that these contributions vanish separately.

Consider first the orbital magnetic moment operator,

$$\mathbf{M}_0 = \beta \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{p}_i, \quad (30)$$

where $\beta$ is the Bohr magneton ($\beta = e\hbar/2mc$). The only properties of $\mathbf{M}_0$ which we need are its being Hermitian and pure imaginary.

Since $\mathbf{M}_0$ does not operate on the spinor parts of the wave function, and since the latter are orthonormal (Eq. (9)) we have

$$(\psi^*, \mathbf{M}_0\psi)$$

$$= \sum_{\substack{\text{all } s_i = \pm \frac{1}{2}}} \int \varphi^*_{s_1 \cdots s_n} \mathbf{M}_0 \varphi_{s_1 \cdots s_n} d\tau_1 \cdots d\tau_n, \quad (31)$$

$$= \sum (-1)^S \int \varphi_{-s_1 \cdots -s_n} \mathbf{M}_0 \varphi_{s_1 \cdots s_n} d\tau_1 \cdots d\tau_n,$$

using the complex conjugate form of Eq. (28). Since we sum over all $s_i$ in Eq. (31), the signs of all $s_i$ can be changed in the last expression to give

$$(\psi^*, \mathbf{M}_0\psi)$$

$$= \sum (-1)^R \int \varphi_{s_1 \cdots s_n} \mathbf{M}_0 \varphi_{-s_1 \cdots -s_n} d\tau_1 \cdots d\tau_n, \quad (32)$$

$$= \sum (-1)^R \int \varphi_{-s_1 \cdots -s_n} \mathbf{M}_0^* \varphi_{s_1 \cdots s_n} d\tau_1 \cdots d\tau_n,$$

(where $R = n/2 + \sum s_k$) using the Hermitian property of $\mathbf{M}_0$. Since, however, $\mathbf{M}_0$ is pure imaginary, $\mathbf{M}_0^*$ is $-\mathbf{M}_0$. Further, $n$ is even so that $\sum_{k=1}^{n} s_k$ is an integer and $(-1)^{\Sigma s_k} = (-1)^{-\Sigma s_k}$. It follows then from Eqs. (31) and (32) that

$$(\psi^*, \mathbf{M}_0\psi) = -(\psi^*, \mathbf{M}_0\psi) = 0, \quad (33)$$

which proves our theorem for orbital-magnetic moments.

The spin magnetic moments must be handled in more detail. The corresponding operator is

$$\mathbf{M}_s = 2\beta \sum_{i=1}^{n} \mathbf{s}_i. \quad (34)$$

The theorem will be proved for the $x$ component of $\mathbf{M}_s$ and similar proofs can be constructed for the other components. It must be shown, then, that

$$\psi^* \cdot \left( \sum_{i=1}^{n} s_{x_i} \right) \psi = 0, \quad (35)$$

where the dot stands for the scalar product of the spinor wave functions. From the matrix representation of $s_x$, Eq. (12), and the explicit form of $\psi$, Eq. (14), it is seen that

$$\left( \sum_{j=1}^{n} s_{x_j} \right) \psi$$

$$= \sum_{\substack{\text{all } s_i = \pm \frac{1}{2}}} \left( \sum_{j=1}^{n} \varphi_{s_1 \cdots -s_j \cdots s_n} \right) \xi_1^{(s_1)} \cdots \xi_n^{(s_n)}, \quad (36)$$

since $s_{xj}$ operates on the spin of the $j$th electron only. Using Eqs. (28) and (36) we have, writing

$$S' = n/2 - \sum_{k=1}^{n} s_k',$$

$$
\begin{aligned}
\psi^* &\cdot (\sum_{i=1}^{n} s_{xi})\psi \\
&= ( \sum_{\text{all } s_{i'}} (-1)^{S'} \varphi_{-s_1'\cdots-s_n'} \xi_1^{(s_{1'})} \cdots \xi_n^{(s_{n'})} ) \\
&\quad \cdot ( \sum_{\text{all } s_i} \{\sum_{j=1}^{n} \varphi_{s_1\cdots-s_j\cdots-s_n}\} \xi_1^{(s_1)} \cdots \xi_n^{(s_n)} ) \\
&= \sum_{\text{all } s_i} (-1)^{S} \varphi_{-s_1\cdots-s_n} (\sum_{j=1}^{n} \varphi_{s_1\cdots-s_j\cdots-s_n}),
\end{aligned}
\tag{37}
$$

where we have used the orthonormality of the $\xi_i^{(s_i)}$. The last sum may now be shown to vanish by pairing off the terms which contain the same $\varphi$ factors. Thus, the coefficient of the term in $\varphi_{s_1\cdots-s_j\cdots-s_n}\varphi_{-s_1\cdots-s_n}$ is the sum of $(-1)$ raised to the powers $(n/2 - \sum s_k)$ and $(n/2 + \sum s_k - 2s_j)$. If one factors $(-1)$ to the power $(n/2 - \sum s_k)$ from this coefficient the remaining factor is one plus $(-1)$ to the power $(2\sum s_k - 2s_j)$. This remaining factor is equal to zero; for $\sum s_k$ is an integer since $n$ is even, and hence $(2\sum s_k - 2s_j)$ is an odd integer. It follows that the coefficient vanishes and therefore Eq. (35) is established.

This completes the proof of the theorem stated at the beginning of the section.

# A Demonstration Experiment on Positron Annihilation

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A lecture demonstration on positron annihilation is discussed. By use of two small Geiger counters and a standard Rossi coincidence circuit, we were able to demonstrate that the two annihilation photons go in opposite directions. A special arrangement of lead shielding is suggested.

THE annihilation of a positron and an electron to give two photons is one of the most striking phenomena of modern physics; so it is desirable, if possible, to demonstrate this effect to elementary physics students. The two-photon annihilation is a fine example of Einstein's relation for the conversion of mass into energy. Further, the observation that annihilation does not occur with the emission of one photon, but demands two photons emitted in opposite directions is an excellent example of the application of the principle of conservation of momentum. (I should remark here that we cannot convince the students in an elementary physics class that the emission of two photons rather than three, or more, should be the predominant process. That only two are usually emitted depends, in essence, on the smallness of the fine structure constant: $e^2/hc = 1/137 \ll 1$.)

Beringer and Montgomery[1] measured the angular correlation between the two annihilation photons, using small (3 cm × 1 cm) Geiger-Müller tubes, feeding into a Rossi coincidence circuit of 3-$\mu$sec resolving time. Recently De-Benedetti *et al.*[2] greatly improved these measurements by use of anthracene crystals and photomultipliers to detect the annihilation radiation. This improvement allowed a much faster coincidence circuit (0.3-$\mu$sec resolving time); and the higher efficiency of the anthracene crystals for detecting $\frac{1}{2}$-Mev annihilation radiation permitted use of much better angular resolution. While the former experiment was able to show that the two photons were emitted within about 1° of exactly opposite directions, the latter experiment measured certain small deviations of the two photons from going in exactly opposite directions. These deviations were interpreted as a change of the photon directions in the laboratory system as a result of motion of the positron and electron prior to annihilation.



FIG. 1. Arrangement of positron source $S$, lead shield, and paraffin slab A.
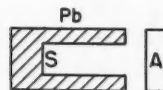
[1] R. Beringer and C. G. Montgomery, Phys. Rev. 61, 222 (1942).

[2] DeBenedetti, Cowan, Konnecker, and Primakoff, Phys. Rev. 77, 205 (1950).
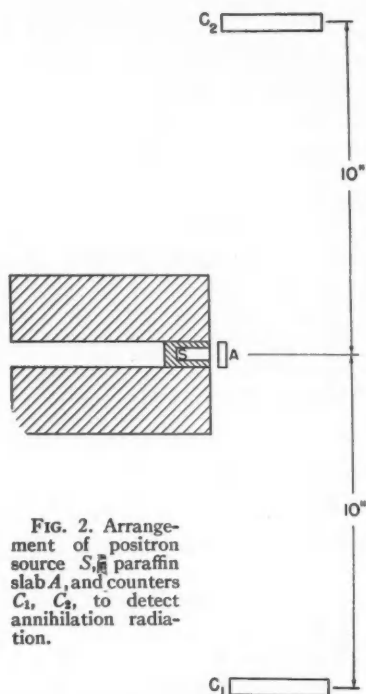
FIG. 2. Arrangement of positron source $S$, paraffin slab $A$, and counters $C_1$, $C_2$, to detect annihilation radiation.

Both of these measurements were made using several mC of 12.9-hr $Cu^{64}$ as the positron source, surrounded by a foil to slow down the positrons, and annihilate them. $Cu^{64}$ is one of the few positron emitters that produces very few nuclear gamma-rays. This great advantage, discussed below, is partly offset by the disadvantage of having to make special arrangements that the $Cu^{64}$ be made to order for the demonstration.

In our demonstration we used a source of 80-day $Co^{56}$ of strength several millicuries, loaned to us by W. Woodward, who was using it in an experiment on positron-electron scattering. For use in demonstrations over a period of years a positron source of longer half-life such as 2.6-yr $Na^{22}$ is suggested. These two positron sources emit many nuclear gamma-rays for each positron that is emitted, so that unless precautions are taken almost all of the observed counting rate will be due to chance coincidences from the nuclear gamma-rays, rather than true coincidences from the annihilation radiation. If scintillation counters and a fast coincidence circuit, as used by DeBenedetti, are available, the demonstration will be much improved. Since we used standard Geiger counters and coincidence circuit, we had to take the special precautions discussed below to reduce the chance coincidence counting rate.

The source $S$ was mounted inside a hollow lead cylinder and covered with thin Nylon, as shown in Fig. 1. A large fraction of the positrons from $S$ were multiply scattered by the lead walls and reached the paraffin slab $A$, where they were slowed down and annihilated. (Any slab several mm thick of low atomic number, which will have small likelihood of absorbing the 0.5-Mev annihilation radiation, serves well as annihilator $A$.) It was possible to shield the counters from nuclear gamma-rays coming directly from the source $S$, without decreasing the counting rate caused by annihilation radiation from $A$. The arrangement used is shown in Fig. 2. The shielding was provided by two lead bricks, each $2 \times 3 \times 6$ in., but better shielding might be advisable. Geiger counters $C_1$ and $C_2$ were brass Ballentine counters, $\frac{1}{2}$ inch in diameter with 3-in. sensitive length. They fed into a standard Rossi coincidence circuit which fed into a register. The appropriate distance between annihilator $A$ and each counter should be determined for the source used. Too small a distance gives a high chance-coincidence rate, while too large a distance decreases the true coincidence rate from annihilation radiation to such a low value that the demonstration takes too long. With the arrangement shown the following data were taken for the class: (a) both counters and the source on the table, 15 counts and 13 counts in successive minutes, (b) one counter raised several inches, so that $C_2$, $A$, and $C_1$ are no longer in line, 4 counts in one minute.

It is desirable for the students to understand that the 4 counts recorded with the counters out of line *could* be chance coincidences. For this purpose it is helpful to demonstrate chance coincidences by placing separate small sources of about 1-$\mu$c strength near each counter, and measuring both the high single counting rates and the coincidence rate. It is also desirable to demonstrate, by a magnetic deflection apparatus, that particles of positive charge are emitted from the source $S$.

This demonstration was performed while the author was at the Cornell University Physics Department. He is grateful to W. Woodward and M. Shapiro for the loan of equipment used in the demonstration.

# The National Science Foundation—Its Organization and Purposes

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The establishment of the National Science Foundation and its anticipated impact on teaching and on basic research in the Natural Sciences are described. Two very important parts of its program are the support of basic research by grants, and the award of fellowships for predoctoral and postdoctoral study.

I WELCOME the opportunity to extend the greetings of the National Science Foundation, as well as my own, to the American Association of Physics Teachers. As one who for years had an active interest in the teaching of physics, it has been very gratifying to see the healthy growth of the Association and the expansion of its membership. In my present position I shall have official reasons for being interested again in the problems which engage the attention of physics teachers and of science teachers generally. As the Foundation is about to enter upon its first program of research support and fellowship aid, I anticipate that there will be many ways in which the understanding and cooperation of the AAPT will be most helpful to us. The Foundation, in turn, hopes that some of the plans which I shall briefly review may prove of interest to you.

Most of you are well acquainted with the history of the long struggle to establish a National Science Foundation, which began even before the war ended and dragged on until May, 1950, when the National Science Foundation Act of 1950 was finally passed. In fact, I should go further than this and say that many of you had an active part in helping to establish the Foundation. Since these matters of history are now familiar to all, I shall not take up any part of today's discussion with a recapitulation, but instead will look ahead a little and try to give you a picture of what we plan to be doing in the next few months.

Most of 1950 was consumed before the President announced the membership of the National Science Board. As finally constituted, the Board of 24 members seems admirably to have lived up to the intent of the Act, which was that it should be broadly representative of the views

of scientific leaders in all areas of the Nation; that its members should be chosen for records of distinguished service; and that they should be eminent in the basic sciences, engineering, agriculture, education, or public affairs. The Board held its first meeting on December 12, 1950, and has been meeting regularly on almost a monthly basis since that date. When I took office as Director on April 6, 1951, the immediate tasks were the organizing and staffing of the Foundation, and the preparation of an operating budget for the first year, fiscal 1952. Although our staff is still far from complete, I am happy to say that key staff members, whose presence is essential to the launching of our major programs, have now been chosen.

In particular, I should like to mention that our Deputy Director is Dr. C. E. Sunderlin, organic chemist and former head of the Office of Naval Research branch office in London. Dr. John Field, physiologist, is on leave of absence as Chairman of the Department of Physiology of the University of California Medical School at Los Angeles in order to serve as Assistant Director for the Division of Biological Sciences, and is, at present, also acting Assistant Director for the Medical Research Division. Dr. Harry Kelly, physicist, formerly scientific and technical advisor to the Supreme Commander for the Allied Powers in Japan, and more recently chief scientist in the ONR Branch Office at Chicago, is Assistant Director for the Division of Scientific Personnel and Education. Of special interest to this Association is the fact that on November 1, 1951, Dr. Paul Klopsteg, who is going on leave of absence as Professor of Applied Science and Director of Research of the Northwestern Institute of Technology, will join us as Assistant Director for the Division of Mathematical, Physical, and Engineering Sciences. In addition to the scientific staff, we have a small

but effective group of people to take care of administrative problems. The Assistant Director for Administration is Mr. Wilson F. Harwood, who came to us from the National Bureau of Standards, where he was Executive Assistant to the Director.

Although the staff is, and will remain, comparatively small, we have already outgrown the limited space afforded by our first headquarters at 901–16th Street in Washington. On the first of October, we moved to pleasanter and more spacious quarters at 2144 California Street, N.W., a former schoolhouse that still reminds one of its former occupants.

With so many favorable elements combining to get the Foundation off to a good start, it came as a distinct shock last August when the House Appropriations Committee, and then the House itself, cut the Foundation's first operating budget by 98 percent—from the $14,000,000 recommended by the President, to the purely nominal sum of $300,000. Between August 20 and October 20, when a bill was finally passed appropriating $3,500,000 for the Foundation, the scientific societies, including the American Institute of Physics, many scientists, educators, and public-spirited citizens were extremely helpful. On the basis of their own firm convictions of the importance of the program of the Foundation to the national welfare generally, as well as to the defense program, they saw to it that these convictions were brought to the attention of Congressmen and Senators. May I say, on behalf of the National Science Board and the Foundation staff, that we are deeply appreciative of all this support. I think it is only fair to note, also, that the Foundation has consistently had many good friends in both the House and the Senate. One evidence of this fact was the amendment proposed by Senator Alexander Smith of New Jersey, with the backing of Senator Paul Douglas of Illinois, Senator McMahon of Connecticut, Senator Margaret Chase Smith of Maine, and Senators Ives and Lehman of New York, to increase the appropriation for the National Science Foundation from $6,300,000, as reported by the Senate Appropriations Committee, to $10,000,00. Although in the end we came off rather less well than we had hoped, the $3,500,000 finally appropriated will enable us to secure a full operating staff and to initiate the major programs

we had planned. We are grateful that there are a number of legislators in each branch of Congress who understand and appreciate the implications of research and education in the natural sciences for our national welfare and security.

What this drastic reduction will mean is, of course, a material scaling down of what was already a modest program of support of research and education. With the funds now available, we plan to allocate approximately $1,500,000 for the support of basic research in biology and medicine, mathematics, the physical sciences, and engineering; $1,350,000 for the establishment of graduate fellowships; and the balance for several purposes—for development of a national policy for the promotion of basic research and education in the sciences, for scientific information problems; for other services; and for support of the National Scientific Register.

Generally speaking, research support will take the form of grants, which seems the method most suitable to the support of basic research. A number of proposals, especially in the field of the biological sciences, have already been received and are being considered. Now that we know we shall have an operating budget, it is expected that many more proposals will be received. The ability of the investigator and the significance of his project will be the principal criteria in the awarding of grants; but, other things being equal, the Foundation will endeavor to award grants on as broad a geographical basis as possible. Investigators associated with small institutions will receive equal consideration with those from large and well-established research centers. Research proposals will be received and considered by the appropriate science division of the Foundation. Each division will be assisted in this task by a Divisional Committee composed of outstanding American scientists in the particular field involved. The Foundation will also employ expert consultants in each field on a part-time basis. The recommendations of the staff in regard to the awarding of grants are subject to the approval of the National Science Board.

For a period of years now, those of us who have been associated with government-sponsored research programs have been aware of the problems that arise in connection with their administration. These problems need constant attention;

and although we may come no closer to perfection than have some of the other agencies with similar responsibilities, we shall profit by their experiences wherever we can. We acknowledge the difficulties associated with the short-term grant or contract, and will consider grants for longer periods, ranging from two to five years, as individual circumstances and the state of our budget warrant.

We are sympathetic, also, with the desire of the scientist to publish over his own name. Investigators receiving Foundation support will be encouraged to publish in the regular scientific journals. Progress-reporting will be held to a minimum, and will be chiefly confined to administrative matters.

As a second major program, plans are also under way for the awarding of several hundred predoctoral and postdoctoral fellowships in the natural sciences for the academic year 1952–1953. Announcements of these fellowships will very shortly be mailed out by the National Research Council, which is assisting the National Science Foundation in this program. You are all well aware of the Council's many years of experience in the administration of fellowships, and it is a great help to us to be able to avail ourselves of their facilities and competence.

Fields of study are limited to the natural sciences, and fellowships will be awarded on the basis of ability only. In the case of candidates with equal ability, as judged from the selection process, account will be taken of geographical distribution. A fellow may attend any accredited nonprofit institution that will accept him. Stipends will enable the student to pursue his studies without the necessity of remunerative employment. Beginning with the second year, family allowances will be included. Postdoctoral fellowships will be awarded under the same general conditions, and stipends will be proportionately larger.

Although the funds presently available will permit the support of only a limited number of fellows, there is every reason to hope and to believe that these fellowships may be the means of making graduate study possible to gifted individuals whose capabilities might otherwise have been less usefully channeled. Part of the interest in and importance of this type of ·endeavor is that the same elements of potentiality

exist in a small-scale effort as in one of greater magnitude. If this fellowship program of the NSF can succeed only half as well as the National Research Fellowships in physics in the 1920's, we shall feel that it has been completely justified.

It is clear from the text of the National Science Act that the major aim of the Foundation is the promotion of basic research and education in the sciences. It is to the latter, education in the sciences, to which I wish now to address a few remarks. There is no need for me to explain to you the importance of this aspect of the support of science. In the field of physics it is essentially the aim of this Association. We cannot have vigorous, healthy research in science without adequate and competent instruction.

The only aspect of this part of the mission of the Foundation, now being taken care of explicitly, is our graduate fellowship program. However, all scientists, I am sure, will agree that research and education really go hand in hand and that the support of basic research in physics, for example, has important and essential educational aspects.

Nevertheless, we are well aware that support of research and education in the graduate schools of our universities will in itself not accomplish all that should be done. Broadly speaking, the problem goes back through the colleges—into the secondary schools, in fact, where instruction in science begins. Although our plans for the current year have of necessity been limited to what may be termed the first and most important phase of this problem, it is abundantly clear that we shall have to give careful study to the broader problem.

The problem of instruction for undergraduates in colleges raises a number of interesting points. If we are to stimulate progress in science, and if we are to try to increase the output of scientists to meet the demands of the current emergency, considerable thought should be given to the best means of stimulating interest in science as a career on the one hand, and on the other, to ascertaining whether more students with high aptitude for science can be found and be given opportunity to go through college and graduate school.

It will certainly be agreed that, while a number

of things may be done to reach prospective students and provide opportunities for study in science, nevertheless, one of the fundamental problems is the teaching of science itself. Several true but trite remarks may be made at this point, such as that we need more and better teachers, and teachers should have higher salaries. There are a few who, like Shakespeare, feel, "To teach a teacher ill beseemeth me." But I believe that a greater number will agree with Browning that "'Tis the taught already that profits by teaching."

The question is how and where can something more be done about the teaching of science. At any rate, any thought which is given to improving educational opportunities in science should pay attention to improving the teaching of science. Much as everyone is in sympathy with doing something about the low relative salaries of teachers, I suspect that increasing salaries alone will not solve this problem.

One of the factors can be identified if one is willing to admit that the profession of teaching is not generally attractive to undergraduates. Research, especially in science, is probably more attractive than teaching to a young boy or girl, but he or she may hesitate to go through the rigorous years of training required, especially if not sure of his aptitude. I think we should also realize that the profession of teaching and research has not had the standing in this country that it has always enjoyed in Europe, for example. What can be done about this? It may be that we are still too close to the frontier days when doing things seemed more attractive than talking about them. One sometimes hears an old, and cynical saying, "He who can, does. He who cannot, teaches."

Let us be frank about another matter. The standing of teaching in the sciences has not always been helped appreciably by the attitude of scientists themselves. In most universities the standing of those devoting their major attention to teaching is apt to be regarded as lower than those who are most productive in research. The existence, however, of this Association proves that this fact has been recognized by the teaching profession in physics and by many of those concerned with research.

Teaching has always had to struggle with the handicap of having little, if any, outside forum. Good research, on the other hand, immediately attracts national and international attention. As a businessman would say, research has sales value, which teaching has not. In view of the admitted importance of teaching in producing the next generations of scientists, it is clear then that special attention should be given to its encouragement.

Let me say at once that when I speak of improving the teaching of science, I am thinking about the teaching of a particular science and not about pedagogical methods in general. It is my view that significant improvement in teaching in a particular subject is most apt to come by concerted effort of the teachers in the subject in question. Thus, improvement in physics teaching will be most effective when undertaken by physicists; chemistry by chemists, and so on.

We must bear in mind certain additional facts in considering this question. There is the striking article from *Science* by Knapp and Goodrich,[1] who analyze the output of scientists throughout our educational institutions. A summary later appeared in the *Scientific American*.[2] You will recall that the 50 institutions of the country which led in the proportion of graduates later receiving doctorates in science, were largely, although not entirely, small colleges. This is a common observation among scientists in universities with large graduate schools. Their experience indicates that most applicants for graduate study tend to come from small colleges and from a limited number of small colleges. Actually, if we distinguish large universities from small colleges by the presence or absence, respectively, of graduate instruction leading to an advanced degree, it is found that the number of bachelors having majored in science who graduate from the large universities exceeds the total number of bachelors in science graduating from small institutions. On the other hand— and this is the important point—the total annual number of Ph.D.'s in science who had their undergraduate instruction in the small colleges exceeds the number who graduated from the large.

[1] Knapp and Goodrich, Science **113**, 543 (1951).
[2] Sci. American **185**, No. 1, 15 (1951).

Taking all these facts into consideration, it is clear that progress may be made along two lines:

(a) Greater recognition and encouragement of undergraduate teaching in the large universities.

(b) Improvement of the instruction in small colleges as a whole so that more of them will graduate young scientists interested in continuing into graduate study.

Although the magnitude and complexity of this general problem gives us pause, perhaps a start may be made in a limited way by the recognition and encouragement of good teaching. This is, of course, one of the aims of this Association, and it is well illustrated by the Richtmyer award. If one accepts the desirability of encouragement and recognition of good teaching, then the question arises: "What form should this encouragement take?" A number of ideas may be considered here. One is the desire on the part of many good teachers in small institutions to keep abreast of modern research in their field and to incorporate this in their teaching. Another is the opportunity, like that provided by this Association and its local sections, for teachers of physics to meet and discuss problems of mutual interest, such as logic of presentation, demonstrations, laboratory experiments, and so on. Still another is the expedient of holding summer school sessions for teachers expressly to discuss their own problems and to hear courses or invited papers from active research workers on research progress in modern physics. Likewise, judicious use of sabbatical years of absence might be profitable.

A few interested institutions might perform the experiment, which is not entirely novel, of aiming graduate courses of study directly at teachers from small institutions, with the award of a Master of Science at the completion of the course. Again, contact with research in physics might well be encouraged by stimulating the performance of research suitable for the facilities of a small institution with the assistance of senior undergraduates. If this is desirable, assistance could be rendered by the staffs of large institutions in the neighborhood.

The National Science Foundation is studying these questions as a matter of policy and will be most appreciative of suggestions along these lines. It is altogether likely that you, as the ones most immediately concerned with problems of this type, may have other ideas and suggestions as to the kind of thing that is needed in this endeavor. I know that I, as well as Dr. Harry Kelly, who is the man chiefly responsible for the educational side of the program of the National Science Foundation, would welcome your ideas and suggestions along these lines.

It is probable that ultimately this type of study might well extend into the secondary schools. Perhaps a most beneficial policy with respect to this entire matter would be for all concerned to take every opportunity to bring these matters to the attention of the proper authorities.

In summarizing the over-all plans of the National Science Foundation for its first year of operations, I have done so in broad general terms, emphasizing those aspects of the program which I thought would be of greatest interest to this group, and merely mentioning others that surely must be considered for significant progress in science.

I should like to comment in closing that we regard this first year of operations as crucial, because in the course of it we must find new ways of demonstrating again the immediate and practical importance to the Nation of a sound body of basic research and a progressive program of education in the sciences.

It is our hope that you, as members of this Association, will take part in this demonstration. You have the general mission of explaining science, and physics in particular, in a form which the laymen will understand. The support of research locally, state-wise, and nationally, rests ultimately on this kind of understanding. The public is genuinely interested in the findings of science, as witness the frequent news items on the subject, but needs to distinguish between fundamental research and the development of weapons and articles of commercial utility. Perhaps most can be accomplished, if we do not forget to stress that scientific research forms a most important part of the culture and philosophy of mankind. It is one of the chief pacemakers of human thought and as such its influence in our destiny can hardly be overestimated.

# On the Combination of the Principles of Area-Moment and Superposition

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(Received May 24, 1951)

The generalized area-moment propositions developed by the author are combined with the principle of superposition to produce an alternative set of generalized area-moment propositions, which are extremely simple and convenient in application. The method is illustrated by treating in detail two standard cases in beam theory. The method can also be applied advantageously to the solution of statically indeterminate beams.

THE application[1] of the generalized area-moment propositions developed by the author[2] can be simplified and, in the case of ordinary loadings, the necessity for integration almost entirely avoided, by combining the principles of superposition and area-moment as follows.

The generalized area-moment propositions take the form:

$$\theta_b = \theta_a + A/EI, \qquad \text{(Prop. I)}$$

$$y_b = y_a + (b-a)\theta_a + Ab^*/EI, \qquad \text{(Prop. II)}$$

where $\theta_a$ represents the slope of the elastic curve at the point corresponding to $x = a$; $\theta_b$ represents similarly the slope at $x = b$; $y_a$ represents the deflection of the elastic curve at $x = a$; $y_b$ similarly the deflection at $x = b$; $A$ represents the area under the moment curve between the limits $a$ and $b$; $b^*$ represents the distance of the centroid of the same area from the line $x = b$;
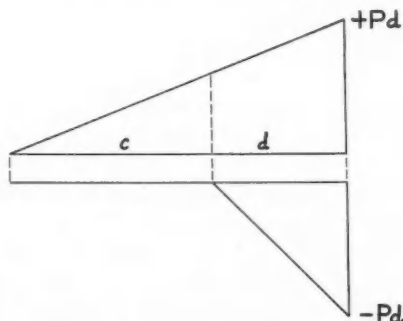


Fig. 1. Separate moment diagrams of forces acting on simple beam with concentrated load placed off center.

[1] A. W. Simon, Am. J. Phys. **19**, 35–36 (1951).
[2] A. W. Simon, Am. J. Phys. **16**, 409–410 (1948).

and, finally, the product $EI$ represents, as usual, the flexural rigidity of the beam considered.

In these equations the area $A$, which represents the area under the *composite* moment curve, can be written:

$$A = \int_a^b M dx = \int_a^b (\sum M_i) dx,$$

where $M_i$ represents the moment produced separately by the force indexed $i$. Transforming the last integral, we have:

$$A = \sum \int_a^b M_i dx = \sum A_i,$$

where $A_i$ represents the area under the *separate* moment curve of the corresponding force.

Similarly, the product $Ab^*$ can be written in the form:

$$Ab^* = \int_a^b M(b-x) dx = \int_a^b \sum M_i(b-x) dx.$$

Transforming the last integral, we have:

$$Ab^* = \sum \int_a^b M_i(b-x) dx = \sum A_i b_i^*,$$

where $b_i^*$ represents the distance of the centroid of the area $A_i$ from the line $x = b$. Hence, as a result of the combination of the principles of superposition and area-moment, the generalized area-moment propositions take the alternative form:

$$\theta_b = \theta_a + \sum A_i/EI, \qquad \text{(Prop. I)}$$

$$y_b = y_a + (b-a)\theta_a + \sum A_i b_i^*/EI. \qquad \text{(Prop. II)}$$
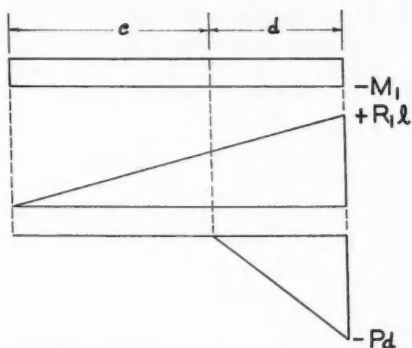
FIG. 2(a). Separate moment diagrams of forces acting on statically indeterminate beam fixed at both ends and with concentrated load placed off center. Moments of forces acting on portion of beam to *left* of section considered.

Additional simplifications can be effected in certain cases by substituting for $\sum M_i$, which represents the moment produced by the forces acting on the portion of the beam to the *left* of the section, its equivalent $-\sum M_i'$, where $\sum M_i'$ represents the moment produced by the forces acting on the portion of the beam to the *right* of the section. Accordingly, we can write:

$$\sum A_i = -\sum A_i',$$

$$\sum A_i b_i^* = -\sum A_i' b_i^{*'},$$

where $\sum A_i'$ represents the sum of the areas under the moment curves corresponding to the forces acting to the *right* of the section, while $\sum A_i' b_i^{*'}$ represents the sum of the products of the same areas and the distances of their respective centroids from the line $x = b$.

The method of application of these principles will be exemplified by treating two standard cases in beam theory.

### SIMPLE BEAM WITH CONCENTRATED LOAD OFF CENTER

Let a concentrated load $P$ be applied transversely to a simply supported beam at a distance $c$ from the left-hand support. Denote the distance of the load from the right-hand support by $d$, and the length of the beam by $l$. The separate moment diagrams are those of Fig. 1.

In order to find the slope $\theta_0$ at the left-hand support, we apply first Prop. II, setting $a = 0$ and

$b = l$, whence we have:

$$0 = 0 + l\theta_0 + (Pdl^2 - Pd^3)/6EI;$$

that is:

$$\theta_0 = -Pd(l^2 - d^2)/6lEI.$$

To find any slope in the range from 0 to $c$, we can now apply Prop. I, which yields:

$$\theta_x = \theta_0 + Pdx^2/2lEI.$$

Substituting in the last equation the value for $\theta_0$ previously obtained, there results:

$$\theta_x = -Pd(l^2 - d^2 - 3x^2)/6lEI.$$

To find the deflection in the same range, we can apply Prop. II again, which now yields:

$$y_x = 0 + x\theta_0 + Pdx^3/6lEI;$$

or, on substituting for $\theta_0$ the value previously obtained:

$$y_x = -Pdx(l^2 - d^2 - x^2)/6lEI.$$

### STATICALLY INDETERMINATE BEAMS

Consider a beam built in at both ends with a single concentrated load $P$ placed at a distance $c$ from the left-hand support. Denote the distance of the load from the right-hand support by $d$ and the length of the beam by $l$. The moment diagrams of the forces acting to the *left*
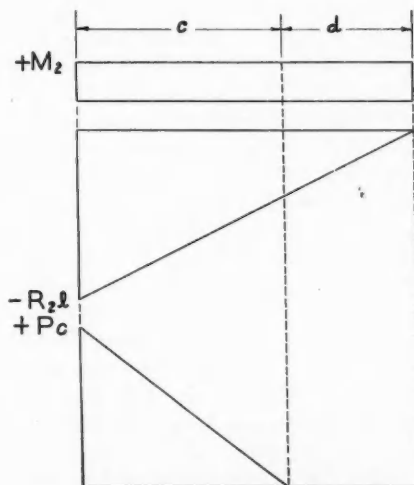


FIG. 2(b). Conditions as for Fig. 2(a), except moments are those of forces acting to the *right* of the section considered.

of the section considered are those of Fig. 2(a), while those of the forces acting to the *right* are those of Fig. 2(b).

If now we apply Prop. I, setting $a=0$ and $b=l$, and utilize the moment diagrams of Fig. 2(a), we have:

$$0=0-M_1l+R_1l^2/2-Pd^2/2;$$

while Prop. II, under the same conditions, yields:

$$0=0-M_1l^2/2+R_1l^3/6-Pd^3/6.$$

These two equations combined with the laws of statics are sufficient to solve the problem of finding the unknown reactions at the supports. However, the problem can be solved also on the basis of area-moment principles alone by utilizing

the moment diagrams of Fig. 2(b) whence we obtain two additional equations:

$$0=0-[M_2l-R_2l^2/2+Pc^2/2],$$
$$0=0-[M_2l^2/2-R_2l^3/3+Pc^2(d+2/3c)/2].$$

By solving these four equations simultaneously the four reactions $M_1$, $M_2$, $R_1$, and $R_2$ can be obtained directly; that is, without reference to the laws of statics.

The illustrative examples given show the advantage in application of the combined area-moment-superposition principles. It will be noted that the formulas for the slope and deflection are obtained practically in one step and in the simplified form in which they are usually given in textbooks on the strength of materials.

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## Microwave Resonance Absorption in Antiferromagnetic Materials*

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(Received August 13, 1951)

Microwave resonance absorption in $Cr_2O_3$, MnO, $MnF_2$, and CoO above and below the paramagnetic-antiferromagnetic transition temperature is reviewed. The amount of the absorption decreases abruptly in lowering the temperature of the sample through its Curie temperature. The nature of the crystal structure changes associated with the changes in magnetic structure are given for comparison. With the exception of CoO, the alignment of magnetic ions to form the antiferromagnetic configuration results in a deformation along an axis perpendicular to sheets of magnetic ions having parallel spins. Similar results are reported for magnetite.

A RECENT trend in magnetism is toward a better understanding of systems containing unpaired electrons whose spin orientations are antiparallel. The spin cancellation may be only partial, as in the case of magnetite and other ferrites, or it may be more nearly complete as in the case of antiferromagnetic materials.

The antiferromagnetic state is considered to be one whose lowest energy requires the maximum number of antiparallel pairs; the exchange energy is considered to be negative. We can think of an antiferromagnetic state, in the simplest terms, as made up of two interpenetrating lattices such that any atom on one lattice has, as its nearest neighbors, atoms of the other lattice and vice versa. One lattice has all of its spins in a given direction while the other lattice

has all of its spins in the opposite direction. We have, therefore, in an antiferromagnetic state, essentially two interlocking self-canceling ferromagnetic systems. The temperature dependence of the magnetization can be considered as the resultant of the temperature dependence of each of these ferromagnetic systems acting independently.

The first method for detecting an antiferromagnetic state came from a study of the temperature dependence of the magnetic susceptibility; then later, by the more direct method of neutron diffraction as introduced by Shull and Smart.[1]

A third approach is through the phenomena of magnetic resonance absorption as it is applied to paramagnetic and ferromagnetic materials—

[1] C. G. Shull and J. Samuel Smart, Phys. Rev. 76, 1256 (1949).

particularly the nature of the change in magnetic resonance absorption as an antiferromagnetic material is taken through its Curie temperature. With the initial work by Griffiths[2] and the theoretical studies by Kittel[3] and Van Vleck,[4] the process of resonance absorption in ferromagnetic systems is, in general, satisfactorily understood. The question now arises as to what happens in a resonance experiment for an antiferromagnetic state.

I will first discuss the resonance absorption of antiferromagnetic systems which will include certain work on magnetite. Following this I will describe the nature of the changes in the crystal structure of antiferromagnetic materials at their Curie temperatures, and then correlate these changes with the resonance experiments.

*Measurements of magnetic resonance absorption in antiferromagnetic materials.*—The available experimental data on the magnetic resonance absorption in antiferromagnetic materials is limited at the present time to the work done at the Naval Ordnance Laboratory by Trounson, Bleil, Wangsness, and myself[5] and some experiments at the University of Chicago by C. A. Hutchison, Jr.[6]

We undertook, in our first experiment, to study $Cr_2O_3$ because it had a convenient Curie temperature that was slightly above room tem-
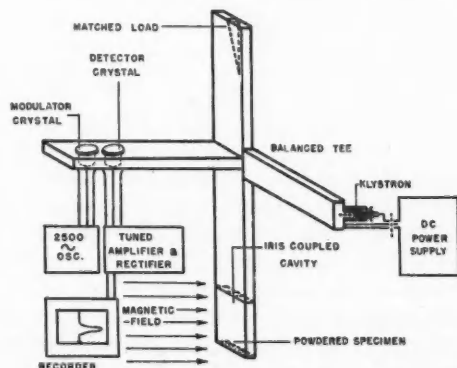


FIG. 2. Resonance absorption curves for $Cr_2O_3$ taken at various temperatures.

perature although its crystal structure was more complicated than other possible materials.

The procedure for measuring the resonance absorption is similar to that used by other investigators for paramagnetic or ferromagnetic materials. Figure 1 shows a schematic diagram of such an apparatus. The sample is placed in the bottom of a rectangular iris-coupled resonant cavity placed in the presence of a steady transverse magnetic field. The absorption is obtained as a change in intensity of reflection from the cavity which was slightly mismatched with respect to the line. As the steady magnetic field is slowly varied, one obtains the relative absorption as a function of the strength of the magnetic field.

Figure 2 shows a set of absorption curves taken for $Cr_2O_3$ at various temperatures. Starting above the Curie temperature we have fairly sharp absorption with increasing peak heights as the temperature decreases until the temperature of the sample is lowered below the Curie temperature in which region the absorption decreases and broadens. In the particular case



FIG. 1. Schematic diagram of magnetic resonance apparatus.

[2] J. H. E. Griffiths, Nature **158**, 670 (1946).
[3] C. Kittel, Phys. Rev. **71**, 270 (1947).
[4] J. H. Van Vleck, Phys. Rev. **78**, 266 (1950).
[5] Trounson, Bleil, Wangsness, and Maxwell, Phys. Rev. **79**, 542 (1950).
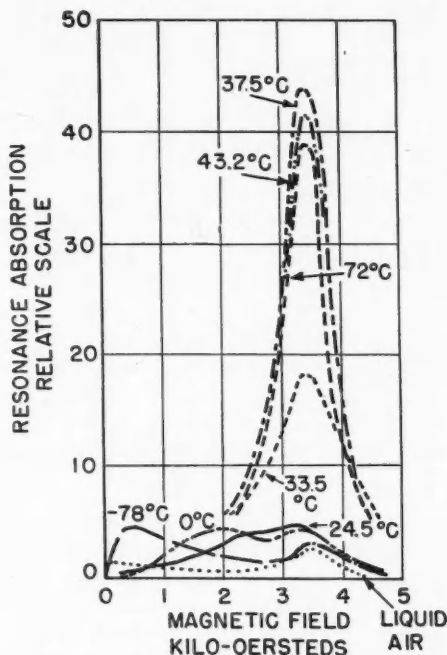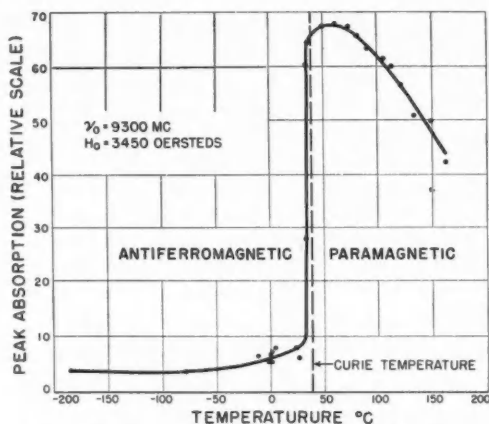[6] By private communication.

FIG. 3. Variation of the height of the resonance absorption of $Cr_2O_3$ with temperature.

illustrated the sample was in a powdered form. It is seen from Fig. 2 that two things happen below the Curie temperature: there is a resemblance of the original peak which gives a $g$ factor near 2, but then a second broad peak develops which moves in toward the origin as the temperature is further decreased. When the material was sintered, however, the absorption below the Curie temperature disappeared entirely. Figure 3 shows the peak height of the curves illustrated in Fig. 2 plotted as a function of the temperature and with relation to the Curie temperature. As the temperature is lowered the peak height increases as the magnetic susceptibility increases; then just below the Curie temperature the peak height drops precipitously to a small residue as shown in Fig. 2—in the case of the sintered material this residue is absent.

We have some information on the behavior of MnO which has a Curie temperature at about
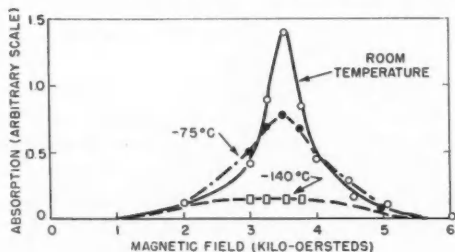


FIG. 4. Resonance curves for MnO taken at various temperatures.

122°K. Figure 4 shows a set of absorption curves that start well above the Curie temperature. These curves represent preliminary data obtained for this material. We find that the peak absorption comes close to that required for $g = 2.0$. It is seen to decrease and broaden with lowering temperature. As contrasted with $Cr_2O_3$, we find the peak height lowering instead of increasing with decreasing temperature in the paramagnetic region. In Fig. 5 there is plotted peak height absorption $vs$ temperature with reference to the Curie temperature. The absorption is reduced appreciably even before the Curie temperature is reached. The exact relationship between the Curie temperature and the resonance absorption requires further investigation.

At the Naval Ordnance Laboratory we have studied other antiferromagnetic materials such as CoO and NiO. It has been difficult to obtain
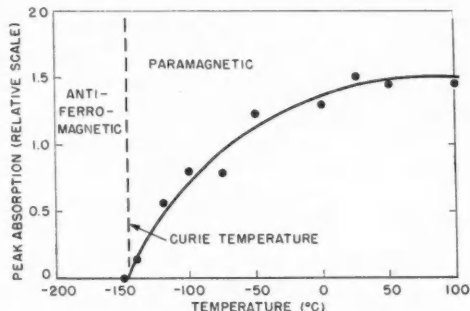


FIG. 5. Variation of the height of the resonance absorption for MnO with temperature.

any appreciable resonance absorption from CoO either below or above its Curie temperature. It has other peculiar properties that I will discuss later. In this connection it is interesting to note that recently Yager, et al.,[7] have had difficulty with resonance measurements in a cobalt ferrite, finding only an extremely broad absorption line.

Clyde A. Hutchison, Jr.,[6] has studied the resonance absorption of a single crystal of $MnF_2$—prepared by Griffel and Stout—from room temperature down to the vicinity of the Curie temperature. Measurements were made with the direction of the high frequency mag-

[7] Yager, Merritt, and Guillaud, Phys. Rev. 81, 477 (1951).

netic field maintained perpendicular to the crystal "*c*" axis (symmetry axis). The "*c*" axis was held both parallel and perpendicular to the static magnetic field. Hutchison's data is shown in Fig. 6. This information gives the results of preliminary measurements, with an estimated accuracy of several tenths of a degree K for the values of the temperatures given. The two sets of curves shown are for the two crystal orientations described above. The surprising fact is the lack of departure between the two curves; the variations shown are attributed by Hutchison as being probably a result of temperature fluctuations. Runs taken at temperatures slightly below 64.4°K showed no trace of absorption. Since the Curie temperature is in the neighborhood of 70°K, we have another case of a sharp drop in the absorption near the transition to antiferromagnetism.
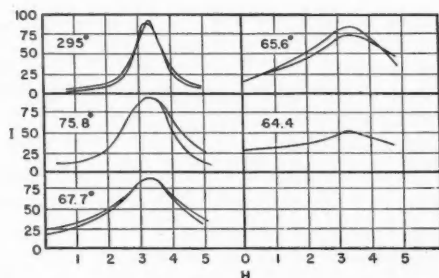


FIG. 6. Resonance absorption curves for a single crystal of $MnF_2$ (temperature in units of degrees Kelvin)—furnished by C. A. Hutchison, Jr.

There are certain resonance absorption experiments performed by Bickford[8] on single crystals of magnetite that I want to mention here in connection with the antiferromagnetic materials. Magnetite has a magnetic transition temperature at about −150°C where there is also an increase in resistance. According to the Néel theory we have in magnetite the trivalent iron ions set antiparallel. It has been suggested that at this transition there is a change from electron disorder to order. Such an explanation was proposed for instance, to account for the increase in specific resistivity.

Figure 7 shows the nature of the absorption found by Bickford[8] for a single crystal of magnetite when the material is cooled through this

[8] L. R. Bickford, Jr., Phys. Rev. **78**, 449 (1950).
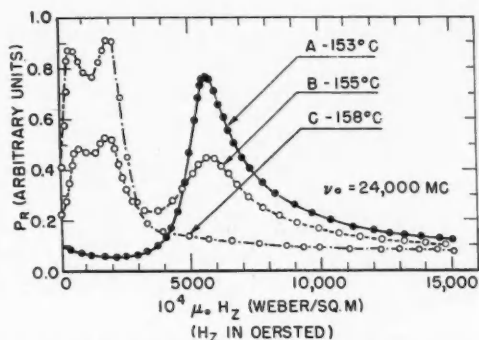


FIG. 7. Resonance absorption for a single crystal of magnetite.

transition temperature. The character of the absorption changed abruptly from −153° to −158°C. The resonance line disappeared while two new peaks appeared at lower values for the steady magnetic field. This type of resonance bears a certain resemblance to that described above for the case of $Cr_2O_3$ with the exception that in this instance the total amount of absorption appears to remain constant while for $Cr_2O_3$ it became greatly reduced below its Curie temperature.

The resonance condition for a ferromagnetic material as developed by Kittel[3] for single crystals is given by:

$$\omega = \gamma\{[H_z + (N_y + N_y{}^e - N_z)M_z] \times [H_z + (N_x + N_x{}^e - N_z)M_z]\}^{\frac{1}{2}}, \quad (1)$$
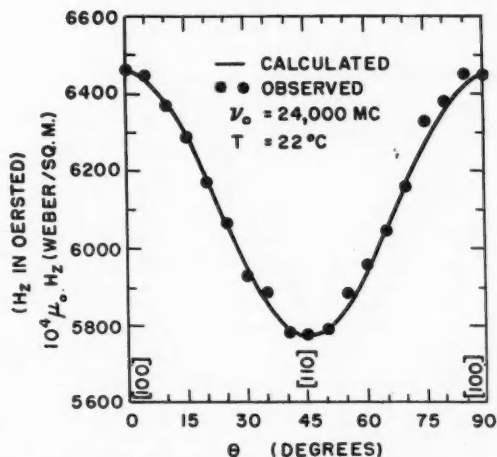


FIG. 8. Resonance induction with respect to crystal orientations in (100) plane of a single crystal of magnetite.

TABLE I. Comparison of magnetic and crystal structure transition temperatures for certain antiferromagnetic materials.

| Compound | Magnetic transition temp. (°K) | Structure transition temp. (°K) |
|---|---|---|
| MnO | 122 | 120[a] |
| FeO | 198 | 203[a] |
| CoO | 292 | 200–300[a] |
| | | 289[b] |
| NiO | ~500 | ~470[a] |
| $Cr_2O_3$ | 322 | 320[b] |

[a] See reference 12.
[b] See reference 13.

where for the 100 plane of the cubic crystal with first-order anisotropy

$$N_z^e = (2K_1/M_z^2)\cos 4\theta$$
$$N_y^e = (\tfrac{3}{2} + \tfrac{1}{2}\cos 4\theta)K_1/M_z^2;$$

likewise for the 110 plane

$$N_z^e = (2 - \sin^2\theta - 3\sin^2 2\theta)K_1/M_z^2$$
$$N_y^e = 2(1 - 2\sin^2\theta - \tfrac{3}{8}\sin^2 2\theta)K_1/M_z^2.$$

The terms used are defined as follows: $\omega$ = resonance frequency; $\gamma$ = gyromagnetic ratio; $M_z$ = intensity of magnetization; $H_z$ = steady magnetic field along the $z$ axis; $N_z, N_y, N_z$ = conventional demagnetization factors; and $N_z^e, N_y^e$ = effective demagnetization factors caused by the presence of magnetic anisotropy.

Equation (1) holds beautifully as shown by Fig. 8 taken from Bickford's work on a single crystal of magnetite at room temperature. The value of the external field at resonance is plotted with respect to the direction in the crystal. The present problem[9] is to develop an analogous theory for antiferromagnetic materials.
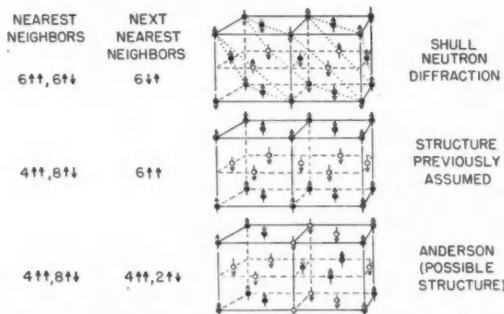
FIG. 9. Magnetic structures for NaCl type antiferromagnetic materials (oxygen not shown).

[9] For a theory of antiferromagnetic resonance published since this paper was presented see C. Kittel, Phys. Rev. 82, 565 (1951).
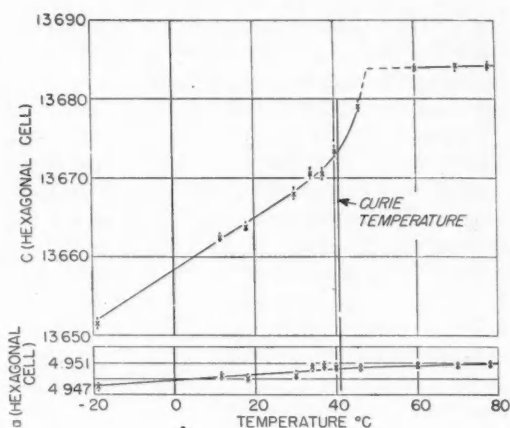
FIG. 10. Variation of length of "$a$" and "$c$" axis with temperature for $Cr_2O_3$.

*Crystal structure changes.*—Within the past year Tombs and Rooksby[10, 11] of the General Electric Company, Ltd.'s Research Laboratories in Britain, and Greenwald and Smart[12] of the U. S. Naval Ordnance Laboratory have been able to find crystal structure changes at the paramagnetic-antiferromagnetic Curie temperatures. It is important to examine the nature of these changes in relation to the magnetic structure as determined by neutron diffraction or to the magnetic susceptibility measurements.

Table I shows a summary of recent structure data for some of the NaCl-type structures and for $Cr_2O_3$. The magnetic transition temperature as determined by susceptibility measurements is compared to the crystal structure transition temperature determined by x-ray diffraction. It is noticed that there is a fairly good agreement between the magnetic and structure transition temperatures. In the case of MnO, FeO, and NiO the cubic symmetry is found to change to

TABLE II. Crystal structure data on magnetite $Fe_3O_4$ obtained by Tombs and Rooksby.

| Temperature | Symmetry | Data based on rhombohedral symmetry | |
|---|---|---|---|
| | | $a_R$ | $\alpha$ |
| 295°K | cubic $a_0 = 8.394 \pm 0.0005$A | $6.434 \pm 0.0005$A | 60° |
| 95°K | rhombohedral | $6.431 \pm 0.001$A | 59°50′ |

[10] N. C. Tombs and H. P. Rooksby, Nature 165, 442 (1950).
[11] H. P. Rooksby, Acta Cryst. 1, 226 (1948).
[12] S. Greenwald and J. S. Smart, Nature 166, 523 (1950).
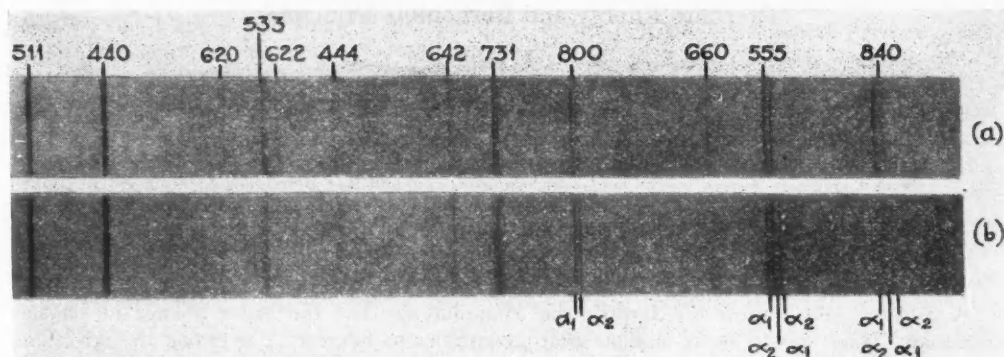
FIG. 11. A portion of x-ray diffraction pattern of $Fe_3O_4$ at temperatures of (a) 295°K and (b) 95°K—furnished by H. P. Rooksby.

rhombohedral symmetry below the Curie temperature in such a way that there is a change in the distance between the 111 planes. CoO, however, changes to tetragonal symmetry.

The relation of the crystal structure change to the spin orientation is indicated in Fig. 9 for FeO, MnO, and NiO. The spins in 111 planes are all parallel and the lattice deformation is along a line perpendicular to the 111 planes.

For $Cr_2O_3$ the space group is $R\bar{3}c$; the structure is like $Al_2O_3$. The chromium atoms having alternate spins form layers perpendicular to the crystal $c$ axis. In Fig. 10 we have plotted the nature of the change in length of the "$c$" axis measured by Greenwald[13] as a function of temperature. A sudden drop in "$c$" is noted near the Curie temperature; the "$a$" axis, however, does not show this abrupt change. The deformation occurs in the direction perpendicular to planes containing the chromium atoms of alternate spin directions.

Tombs and Rooksby[14] have recently obtained some new x-ray diffraction data dealing with the low temperature transformation in magnetite. Figure 11 is a reproduction of an x-ray diffraction pattern showing line-splitting in going from room temperature to liquid air temperature.

[13] S. Greenwald, Nature (in press).
[14] N. C. Tombs and H. P. Rooksby (private communication).

Table II gives the results of their analysis. Again we find a cubic symmetry going into a rhombohedral symmetry with an expansion along a line perpendicular to the 111 planes. Tombs and Rooksby interpret this as showing that the iron atoms, whether in the position of 4-fold or 6-fold coordination positions, are arranged in sheets parallel to the 111 planes and arranged in successive planes with spins antiparallel. They draw an analogy between magnetite and antiferromagnetic materials.

*To summarize we have:*

1. Microwave resonance absorption measurements of antiferromagnetic materials show either broad or no absorption.

2. Lattice deformations occur for antiferromagnetic materials when cooling through the Curie temperature; the greatest change occurring in a direction perpendicular to the sheets of magnetic ions.

3. There is a need for more experimental data on microwave resonance absorption by antiferromagnetic systems, particularly for single crystals and at higher microwave frequencies.

4. The mechanism responsible for magnetic resonance in antiferromagnetic systems is not yet fully understood.

# Pressure Energy and Bernoulli's Principle

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(Received September 4, 1951)

An effort is made to show that the concept of pressure in an incompressible fluid as energy per unit volume possessed by the fluid is an erroneous one, and that Bernoulli's equation is not to be interpreted as a statement of conservation of energy, but rather as a relation between pressure, velocity, and height in the gravitational field.

IN textbooks of physics one often reads the statement under dynamics of fluids, that when an incompressible liquid, of which water may be taken as a sufficiently good example, is subjected to a pressure $p$, then the fluid has, by virtue of that pressure alone, an energy per unit volume represented by $p$, and this in addition to any kinetic energy or gravitational potential energy which it may otherwise have. Having accepted this view, the authors of these books then consistently conclude that Bernoulli's principle, $p + \frac{1}{2}\rho v^2 + \rho g h = a$ constant, is to be interpreted as an example of the conservation of energy, and they assert that the total energy of a unit volume of the liquid, as it moves along a streamline, remains constant. In examining more than thirty textbooks, of which most were of the grade of general physics, but a few more advanced, I found that more than half of them took the viewpoint described above.

If pressure does give rise to the energy $p$ per unit volume, and if Bernoulli's theorem does express conservation of energy per unit volume, then it would seem that any ordinary textbook should scarcely fail to mention such important facts; while if the view is wrong, then it is rather

humiliating that the exact science of physics continues to tolerate or overlook the erroneous statements.

Articles have been written by Kennard[1] and by Van Lear[2] showing that the pressure $p$ should not be looked on as energy density, but in spite of these efforts the claim persists that we give energy to a liquid just by exerting pressure on it, without changing its volume. I shall attempt to give some further reasons and illustrations why the so-called "pressure energy" does not exist, by showing to what impossible conclusions such an idea leads.

A common argument for the existence of pressure energy runs as follows: In Fig. 1 water is pumped into a tank by means of a piston under a pressure $p$. The work done per unit volume pumped in is $p$. If the water is allowed to run out of the tank from an orifice at a level where the pressure is the same, then the energy is received back again in the form of kinetic energy of the issuing stream. Therefore, we are told, the liquid must have this potential energy per unit volume while it is in the tank under the pressure $p$.

It is very easily shown that the work done in pumping the water in simply goes into gravitational potential energy. Suppose the small layer directly under the dotted line at the top of the figure is a unit volume. Then the change resulting from pumping a unit volume into the tank is that we have a unit volume spread over the surface which was not there before. The gravitational potential energy of this layer is $\rho g h$, where $\rho$ is the density of the liquid. Thus the increase of gravitational potential energy of the liquid in the tank is $\rho g h$. But $p = \rho g h$. Hence the work

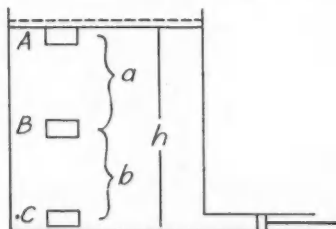

FIG. 1. Tank supplied with a piston and containing water to a height $h$. The dotted line at the top indicates the position of the surface when an additional unit volume of water has been forced in by the piston. $A$, $B$, and $C$ are positions of a solid immersed to different depths.

[1] E. H. Kennard, Science, September 11, 1925, p. 243.
[2] G. A. Van Lear, Am. Phys. Teacher, 2, 99 (1934).

done by the piston is just equal to the increase in gravitational potential energy of the entire body of water and there is no work left over to produce any pressure energy. Those who believe in pressure energy admit, of course, that the liquid has gravitational potential energy proportional to the elevation above the chosen zero level, but they say there is also an energy due to the pressure, so that the unit volume at $B$, for example, has gravitational potential energy $\rho g b$, and pressure energy $\rho g a$, making a total of $\rho g h$, and every unit volume in the tank at any level is said to have the same total energy.

If we consider the total energy of the entire tankful of water of uniform horizontal cross section, the gravitational energy is $\frac{1}{2}Mgh$, where $M$ is the total mass and the pressure energy would also be $\frac{1}{2}Mgh$, a total of $Mgh$. The work required to fill it is only $\frac{1}{2}Mgh$, thus we obtain two units of energy for every one expended in work. By constructing a tank of the proper shape, for example, one of very large horizontal dimensions, small height, the top closed except for a connecting pipe of small diameter extending upward to a considerable height, one might obtain a "pressure energy" any desired number of times the work required to fill the tank. In each case it may be shown that the work done in pumping in the water is just equal to the increase in gravitational energy, while "the pressure energy" may be made any value, at will, by changing the shape of the tank, or by merely rotating the same tank about a horizontal axis through its center of gravity, an operation which requires no work. The limiting case of this violation of the work-energy relation would be that of a tank filled with water and entirely closed except for the piston to which a force could be applied, thus producing a pressure throughout the liquid, and a variable amount of "pressure energy" up to the limit of strength of the tank, without doing any work whatever. What reason can there be for labeling such a quantity energy? Pressure in a fluid is no more energy density than any other force per unit area is.

An argument in favor of the existence of pressure energy, which seems to have been presented seriously in some textbooks, has been brought to my attention. The argument points out that, if we have a piece of cork at $A$ in Fig. 1 and push

it down to the point $C$, we must do a certain amount of work, the cork tends to rise again, and since it has been immersed all the time there is no question of displacement of the liquid entering in during the operation of pushing the cork downward. The argument ignores the fact that unless the cork has a density zero, the work done per unit volume of the immersed body is not equal to the difference in pressure between the points $A$ and $C$; if the immersed body had the same density as the liquid, no work would be done; and if the body were heavier than water, the work would be negative. Further, if $v$ is the volume of the immersed body and $\rho_s$ its density, while $\rho_l$ is the density of the liquid, then the change in the gravitational potential energy of the solid in moving from $A$ to $C$ is a decrease of $\rho_s vh$. The change in the liquid is that a volume $v$ has been lifted from $C$ to $A$, representing an increase in gravitational potential energy of $\rho_l vh$. The net change of gravitational potential energy of liquid and solid is an increase of $(\rho_l - \rho_s)vh$. This is exactly equal to the work done in pushing the solid down from $A$ to $C$, so that the work done is just equal to the gain in gravitational potential energy of the system, and again there is no basis for any energy due to the pressure.

Bernoulli's equation $p + \frac{1}{2}\rho v^2 + \rho g h = c$ states a relation between the pressure and the energy (kinetic and potential) per unit volume of an incompressible and nonviscous fluid in a steady state of flow, for all points along a streamline. It is claimed by many authors that the equation asserts that the total energy per unit volume of the fluid is a constant. The basis for this claim of constant energy is usually that pressure represents energy per unit volume and therefore the sum of the three · terms represents the total energy. If pressure is not energy per unit volume, then this interpretation of the equation, as an expression of conservation of energy, collapses immediately, and the equation is seen to be, aside from the gravitational energy, simply a relation between pressure and kinetic energy per unit volume.

Here again an argument has been offered which at first glance may seem to have weight, but which on closer examination also turns out to be invalid. It runs as follows: The pressure $p$ is a function of position only, since it varies

from point to point, but not with time. The force is $-dp/ds$ where $ds$ is an element of distance along the stream line. Thus it is said that we have a function, $p$, of position only, whose negative gradient is the force acting, and this justifies us in calling $p$ a potential energy per unit volume.

The thing that is wrong with this argument is that the pressure is not a function of position alone. It is a function of the velocity. To make this assertion clearer let us omit the gravitational field. Nothing essential to the problem is changed thereby, for the existence of gravitational potential energy is admitted by all. An equivalent simplification would be to consider a horizontal streamline. Along this line the gravitational energy is constant and may be cancelled from both sides of the equation, thus leaving the equation in the form $p + \frac{1}{2}\rho v^2 = c$. If we know the constants $\rho$ and $c$, where $c$ may be taken as the value of the left-hand side of the equation at some particular point along the streamline, then if the velocity $v$ is given at any other point, we may, at once, without knowing anything more, find $p$. This is equivalent to saying that $p$ is a function of $v$. When $\rho$ and $c$ are given, $p$ may not be determined by giving the coordinates of another point. It is true that $p$ has different values at different points in the fluid, because for a steady flow we could not have different values of $p$ at the *same* point, even at different times, but this does not indicate that $p$ is a function of space alone. Therefore, since $p$ may be determined by giving $v$ as the one other variable, but not by giving coordinates, it cannot be said that $p$ is a function of position alone. It is rather a function of the velocity.

Consider now Bernoulli's equation as it is often obtained by integrating Euler's hydrodynamical equations after having multiplied them through by the element of distance. If the integration is performed between two points where the velocity and pressure are $p_a$ and $v_a$, $p_b$ and $v_b$, respectively, we have the common form, again omitting the part due to gravitational potential energy, $p_a - p_b = \frac{1}{2}\rho v_b^2 - \frac{1}{2}\rho v_a^2$. The left-hand side represents the work done on the unit volume by the resultant nonconservative forces of the surrounding fluid as the unit volume moves from the point $a$ to the point $b$. By virtue of Newton's second law, from which Euler's equations arose, this work per unit volume has been expressed in other terms, namely in terms of the mass per unit volume and the velocity. It is the familiar work-energy equation, applying to solids as well as to liquids. It may be written in the form $W_1 - W_2 = E_2 - E_1$, where the order of the subscripts has become reversed in the first member because the pressure increases in the opposite direction to the motion if the velocity is increasing. The quantities $W_1$ and $W_2$ have the dimensions of energy density, but they do not represent energy possessed by the body. Their difference is shown mathematically to be equal to the difference between $E_2$ and $E_1$, which are called energies of the body for familiar reasons; they represent all the energy the body has under the reference system agreed upon. Nothing should be added to $E_1$ or $E_2$ to obtain the total energy at the points $a$ and $b$ under consideration. If we interpret Bernoulli's equation as expressing the conservation of energy of an element of the fluid, we are asserting that when a unit volume of the fluid is acted upon by nonconservative forces exerted by neighboring portions of the fluid, thereby doing work on the unit volume while moving a certain distance, the mass contained in the unit volume of the fluid has no more energy than before. This is a violation of the work-energy relation.

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### 1952 Summer Meeting of the Association

# The Adsorption of Gases on Metal Filaments, Films, and Single Crystals

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The paper presents the concepts involved in the study of adsorption of gases on solids. Particular stress is placed on the problem of uniform surfaces. An historical study follows which is specifically concerned with (1) metal filaments and evaporated films and (2) surfaces of metal single crystals, both considered as approximations to ideal surfaces for adsorption measurements.

THE adsorption of gases on solids has been studied experimentally and theoretically for many years. This study is important, not only because of the practical applications of the knowledge of the behavior of gases at the surfaces of solids in problems of catalysis and corrosion, but also because it is a physical phenomenon of scientific interest. The purpose of the present paper, which is intentionally restricted in scope, is to outline briefly some of the concepts involved in these investigations and to describe some of the experiments conducted with relatively homogeneous surfaces.

Adsorption is the process by which gas molecules in the presence of the surface of a solid become attached to that surface. It is customary to call the gas in question, the adsorbate, and the solid, the adsorbent. The molecules of the surface of a solid are not subject to balanced forces as are the internal molecules. The unbalanced forces at the surface give rise to surface tension and hence to surface free energy. If a gas molecule approaches the surface, it may be held by the unbalanced forces, reducing some of the surface tension and hence lowering the free energy. Since a decrease in free energy is a transition to a more stable state, adsorption will occur on a solid surface exposed to a gas whose molecules are attracted by the molecules of the solid.

If gas is admitted at a definite pressure and at a definite temperature into a vacuum chamber containing the adsorbent, the amount of gas adsorbed, when equilibrium is established, will depend on the particular adsorbent and adsorbate, and the temperature and pressure of the system. Thus for a given system, it is convenient to present the data in the form of isotherms, showing the amount of gas adsorbed at constant temperature as a function of the pressure, or in the form of isobars, showing the amount of gas adsorbed at constant pressure as a function of the temperature.

It has been pointed out that adsorption results in a decrease in the free energy of the system. Since a gas molecule on the surface of the solid possesses less freedom of motion than a molecule in the gaseous state, adsorption results in a decrease in the randomness and hence the entropy of the system. The heat change (in terms of free energy, entropy, and temperature)

$$dH = dF + TdS \tag{1}$$

is then negative, and one may conclude, in general, that all adsorption processes are exothermic. If a small amount of gas, represented by $dx$, is adsorbed, then the amount of heat evolved may be represented by $dQ$. The differential heat of adsorption is defined as

$$q = dQ/dx, \tag{2}$$

and the integral heat of adsorption for a total amount of adsorbed gas $x$ is defined as

$$Q = \int_0^x dQ = \int_0^x q\, dx. \tag{3}$$

If either $Q$ or $q$ is known as a function of the amount of gas adsorbed, it is possible to find the other, but the expression "heat of adsorption" in the following discussion will refer to the differential heat of adsorption.

It has been found experimentally that in some cases the heat of adsorption is of the order of the heat of condensation of the gas involved, while in other cases the heat of adsorption is one or more magnitudes larger or of the order of the heat evolved in chemical combinations. It is further observed that the former process is re-

versible with respect to pressure and temperature, while the latter is irreversible. Because condensation is traditionally a physical phenomenon, it is usual to call the reversible process physical, or van der Waals' adsorption, and the irreversible process, because of its resemblance to some chemical reactions, chemical adsorption or chemisorption.

The forces involved in the two types of adsorption are, as one might expect, van der Waals' forces in the case of physical adsorption and similar to chemical forces in the case of chemisorption. If one keeps these ideas in mind certain other properties of the two types become plausible. Since van der Waals' forces also attract molecules or atoms of the same kind, we would expect that after one layer of molecules had adsorbed on the surface of a solid, another layer might adsorb on the first layer, the number of layers increasing as the saturation pressure of the gas is approached. This is indeed the case, and one of the properties often associated with physical adsorption is the formation of multilayers of gas molecules. Furthermore, van der Waals' forces can act between any two molecules and one would not be surprised to learn that the adsorption characteristics between an adsorbent and adsorbate depend largely on the adsorbate. Of course different adsorbents act differently, but physical adsorption is relatively nonspecific with respect to the adsorbent. On the other hand chemical forces depend strongly on the particular atoms involved, and one might therefore expect that with chemisorption one would find the adsorbed layer limited to one molecular thickness, and further that the adsorption characteristics depend greatly on both adsorbent and adsorbate. Again this is what happens, and one finds that chemisorption gives rise to monomolecular layers and that the process is highly specific with regard to the adsorbent for a given adsorbate. Summarizing, one may say it is possible to distinguish two types of adsorption, which may be studied under equilibrium conditions by measuring the amount of gas adsorbed as a function of pressure and temperature, and by measuring the heat evolved in the process.

Most measurements of the amount of gas adsorbed have been made by either volumetric or gravimetric methods. The volumetric method may be described in the following manner. A known amount of gas is admitted to a known volume containing the adsorbent, and the pressure is observed. Since the amount of gas, the free volume, and the temperature are known, it is possible to calculate what the pressure would be if there were no adsorption. As the observed pressure is lower than the calculated value due to the adsorption process, the amount adsorbed is easily determined. In the gravimetric method, the adsorbent is weighed by a very sensitive balance, and the increase in weight due to the adsorbed gas is measured directly. Either $q$ or $Q$ may be measured directly by calorimetric methods, or $q$ may be evaluated from curves of the pressure of adsorption as a function of the temperature of adsorption for a constant amount of adsorbed gas by an application of the Clausius-Clapeyron equation.

In order that the amount of gas needed to cover the surface of the adsorbent shall be an appreciable fraction of the amount occupying the free volume in the volumetric method, or have an observable weight in comparison to the weight of the adsorbent in the gravimetric method, adsorbents are frequently used with great surface area per unit volume or weight. Thus measurements have been made on porous or granulated adsorbents such as charcoal, catalysts, evaporated films, and powdered materials. While measurements of this type have provided a wealth of practical and theoretical information, very little can be said about the details of the process of adsorption, because the surface of such materials is almost unknown. Adsorption is a surface effect, and a complete understanding of such a phenomenon involves a complete knowledge of the surface of the adsorbent. On the type of adsorbents mentioned, it is certain that the surfaces are not uniform so that their adsorbing characteristics vary from place to place over the surface. It is even difficult to obtain the value of the effective area. This non-uniformity can be represented by considering the heat of adsorption to vary over the surface. Thus experimentally one often finds that the heat of adsorption decreases as the surface becomes covered with gas molecules, because those regions with high heat of adsorption acquire gas more easily. Furthermore, a heterogeneous sur-

face may be recognized by the behavior of the isotherms with temperature. Many isotherms, particularly in cases of chemisorption, show a saturation amount; that is, after a certain pressure has been reached, no further gas can be adsorbed. If the heat of adsorption varies with different regions of the surface, this saturation amount will decrease as temperature increases, because the regions of low heat of adsorption will be unable to retain the gas molecules at higher temperatures. Even though one may observe the presence of this nonuniformity, one measures, by the above methods, the average adsorption over many different, small, adsorbing surfaces whose area and nature are unknown. These considerations are generally appreciated, and consequently, there have been repeated efforts to study adsorption on the most uniform surfaces possible.

With these ideas in mind, the writer presents a summary of some of the experiments conducted on metal adsorbents. The studies chosen are those in which particular care was taken to obtain as uniform a surface as possible. In so doing, the writer is not discussing the great majority of adsorption history, nor even phases which predominate in the literature today. This is not done out of lack of appreciation for the importance of the work, apparently ignored, but simply because in so vast a field as adsorption there is value in considering one aspect of the entire problem.

An ideal experiment in adsorption would be to determine isobars, isotherms, and the heat of adsorption as a function of adsorbed gas, on a completely known surface. By a completely known surface is meant a surface of which the nature and position of every atom is known. A first approximation to this ideal for metals can be found in the aged surfaces of wire filaments and in carefully evaporated metal films. Probably the second approximation is one face of a metal, single crystal. Measurements have been made on surfaces of these types, and a brief description of some of the more outstanding studies follows.

## ADSORPTION ON FILAMENTS AND EVAPORATED FILMS

Among the first and most important experiments on relatively well-known surfaces are those performed by Irving Langmuir[1] around 1915. Using a volumetric method with great precision, Langmuir studied the behavior of gases in the presence of hot tungsten filaments. These experiments caused Langmuir to propose a change in the previous picture of adsorption. Theories of adsorption up to this time supposed that adsorbed gas was in the form of a compressed film near the surface of the solid, but Langmuir's experiments led him to formulate a monolayer theory of adsorption leading to the famous Langmuir isotherm.

Langmuir found that on a tungsten filament at 3300°K in the presence of oxygen, 50 percent of the molecules that struck the tungsten formed $WO_3$, which evaporated and condensed on the glass bulb, but since there are three oxygen atoms in $WO_3$, and only two in $O_2$, he argued that the tungsten was at least partially covered with oxygen. Since $WO_3$ distills rapidly from a tungsten filament at 1200°K, the oxygen remaining on the surface at 3300°K was not in the form of $WO_3$. Still more interesting results were obtained when both oxygen and hydrogen were admitted to the bulb. With the filament at 1500°K, the oxygen reacted as though no hydrogen were present. The hydrogen remained inert until the oxygen was completely adsorbed, although in a tube containing a hot tungsten filament hydrogen ordinarily forms atomic hydrogen which will reduce any $WO_3$ on the walls of the bulb. After the oxygen had adsorbed, the hydrogen suddenly reacted with the $WO_3$. Langmuir concluded that (1) the oxygen practically covered the surface of the tungsten in a layer only one molecule deep, (2) the adsorbed oxygen would not react with the hydrogen, and (3) the layer prevented the formation of atomic hydrogen until the oxygen was adsorbed and vacant area could be reached by the hydrogen. Langmuir pointed out that such results were not possible if the oxygen had formed a compressed layer on the tungsten, as then the hydrogen would react with oxygen, nor was it possible that the adsorbed layer consisted of $WO_3$, as this compound will react with molecular hydrogen at temperatures as low as 500°K. The results are explained, however, if one pictures a monomolecular layer of oxygen

[1] I. Langmuir, J. Am. Chem. Soc. 38, 2221–2295 (1916), esp. pp. 2267–2278.

held by chemical bonds to the body of the tungsten.

Using the picture presented by Bragg of the regular arrangement of atoms in a crystal, Langmuir[2] imagined the surface atoms forming an array of definite sites where the gas molecules or atoms might be held. He pictured a gas molecule being caught and after a short time, evaporating, the adsorption being caused by the time lag between condensation and evaporation. Because of the short range of the forces of the solid, only one layer of adsorbed molecules can form. Assuming that the adsorbed molecules did not react with each other, and that each site could have only one molecule, Langmuir derived an expression from kinetic theory for the amount of gas adsorbed at a given pressure and temperature. This expression, known as the Langmuir isotherm equation, is

$$\theta = Bp/(1 + Bp), \qquad (4)$$

where $\theta$ is the fraction of the total available sites occupied, $B$ is a function of the temperature for a given adsorbent and adsorbate, and $p$ is the pressure of the gas. Under the same assumptions Langmuir derived modifications of his theory to allow for dissociation of the adsorbed molecules and for the presence of several types of sites on the same adsorbent.

The above expression (4) is one of the most important in the study of adsorption, and it can be derived under a number of different assumptions regarding the mechanism of adsorption. The picture of the chemical bonding means that the theory is most accurate in the case of chemisorption, but a great deal of physical adsorption data fits this isotherm equation. Langmuir intended his theory to apply to homogeneous surfaces, and he tried to test his theory on mica, cover glasses, and platinum foil.[2] The results can today be identified with physical adsorption. Furthermore, a variation in the total number of sites with temperature was observed, a characteristic of nonuniform surfaces. For present purposes it is important to note that it was Langmuir's studies on tungsten filaments which led to the monolayer concept of adsorption including the idea of sites and the Langmuir isotherm.

[2] I. Langmuir, J. Am. Chem. Soc. **40**, 1361–1403 (1918).

Following Langmuir's work a great many experiments were performed with filaments, particularly in the study of thermionic emission.[3] A discussion of these efforts will not be entered into here, but the use of filaments was extended in a striking manner by J. K. Roberts. In a series of researches, both experimental and theoretical, he studied the formation of gas films on single tungsten filaments.

Roberts[4,5] studied adsorption of gases on solids by observing the behavior of the Knudsen accommodation coefficient. If the surface of a solid is maintained at a temperature $T_2$ slightly above the temperature $T_1$ of the surrounding gas, the gas molecules striking the solid rebound with an energy corresponding to some temperature $T_2'$. If one defines the accommodation coefficient as

$$\alpha = (T_2' - T_1)/(T_2 - T_1), \qquad (5)$$

it can be shown that $\alpha$ depends upon the mass of the atoms on the surface of the solid. Roberts measured $T_2$ by the resistance of the filament and $T_2'$ by the rate at which heat was carried away from the filament. The temperature $T_1$ of the gas was taken as being the same as the temperature of the glass envelope. After the filament was flashed to 2000°K in a high vacuum, neon was admitted and $\alpha$ observed. Hydrogen was then admitted and the presence of adsorbed hydrogen was detected by the change in $\alpha$. Roberts observed that hydrogen was irreversibly adsorbed in a monolayer in about ten minutes, both at room temperatures and at liquid air temperatures at pressures of the order of $10^{-4}$ mm of Hg.

Roberts extended his work to a study of the heat of adsorption as a function of the amount of gas adsorbed. By using the wire itself as a calorimeter, Roberts observed the heat of adsorption while measuring the pressure of the gas with a Pirani gauge. The heat of adsorption of hydrogen on tungsten was found to decrease about linearly from 45,000 cal/mole to 18,000 cal/mole during the formation of a monolayer.

A decrease in heat of adsorption with amount of adsorbed gas had been noted in other experi-

[3] J. H. de Boer, *Electron Emission and Adsorption Phenomena* (Cambridge University Press, London, England, 1935).
[4] J. K. Roberts, Proc. Roy. Soc. (London) **152A**, 445–447 (1935).
[5] J. K. Roberts, *Some Problems in Adsorption* (Cambridge University Press, London, England, 1939).

ments and H. S. Taylor had explained the fact by assuming a heterogeneous surface. The work of Langmuir[6] and Johnson[7] indicated, however, that the surface of an aged tungsten filament consists of (110) and (100) crystal planes, and Roberts felt that Taylor's explanation could not apply. If, however, the molecules of gas were mutually repulsive,[8] then as the surface becomes covered one would expect succeeding molecules to be repelled by the molecules already adsorbed and the heat of adsorption would decrease. Roberts assumed the adsorbed gas could form in a monolayer in which the molecules either were situated on definite sites as in Langmuir's picture, or were in the form of a two-dimensional gas. If the molecules were immobile, then on the average the fraction of the number of neighboring sites of any particular vacant site which were occupied would be the fraction of the total number of occupied sites, $\theta$. Then the heat of adsorption would decrease about linearly from the initial amount $q_i$ to $q_i - ZV$, where $Z$ is the number of nearest neighbors of any particular site and $V$ is the interaction energy between a pair of molecules separated by the distance between sites. If, on the other hand, the molecules formed a two-dimensional gas or were mobile, the approach of another gas molecule toward the surface, on which some gas molecules were already adsorbed, would force the adsorbed molecules away until the interaction was negligible. Thus as the surface became covered, there would be no decrease in the heat of adsorption until the surface was half covered, at which point it would no longer be possible for the adsorbed molecules to separate themselves without interaction, and for $\theta > \frac{1}{2}$ Roberts predicted a sudden drop in the heat of adsorption. According to this point of view, Roberts interpreted his results as meaning that the adsorbed hydrogen on tungsten formed an immobile monolayer.

The above picture has been refined by Roberts and Miller[9] by taking into account the fact that the equilibrium position of atoms in either type of layer is modified by the interaction energy. If this is done, one finds, unfortunately, that the difference in behavior of the heat of adsorption in the two cases is less than the experimental error.

Roberts also studied the formation of oxygen films on tungsten, using the same methods as with hydrogen. In his earlier papers Roberts[4] reported the formation of two layers of oxygen on tungsten. On clean tungsten the first layer was pictured as being atomic, two adjacent sites being needed to adsorb a single oxygen molecule. By means of a statistical experiment with a model surface, and later by statistical theory, Roberts showed that a complete monolayer would leave gaps or holes and that only about 92 percent of the total number of sites would be occupied. The heat of adsorption for this first atomic layer was measured as 134,000 cal/mole. A second layer, which seemed to be molecular, apparently formed in the holes left in the first layer with a heat of adsorption of 48,000 cal/mole. In later experiments with the accommodation coefficient, Roberts[5] came to the conclusion that oxygen formed in three, rather than two, steps. The first film is stable, chemisorbed, and is pictured as molecular, not atomic, each molecule preventing stable adsorption on the nearest neighboring sites. The second film is adsorbed in the gaps left by the first film, the number of sites available being 20 percent of the sites available to the first film. Still a third film is pictured by Roberts, as a second layer, forming over the first layer completed in the two steps.

The work of Roberts is important because it is a study of adsorption on extremely simple systems. Using a relatively homogeneous metal surface, which could be cleaned far better than most, the experiments approximate adsorption on a known surface. Furthermore, the experiments were carried out in an attempt to study the detail of the adsorption process itself from a physical point of view, and although his theories were perhaps extended beyond the point of experimental verification, there was a closer

[6] I. Langmuir, J. Am. Chem. Soc. **54**, 2798–2832 (1932), esp. p. 2830.

[7] R. P. Johnson, Phys. Rev. **54**, 459–467 (1938).

[8] In order that the forces between adjacent atoms or molecules may be repulsive, they must be held on their sites by forces other than van der Waals' forces. This means that mutual repulsion between adsorbed molecules would occur more often with chemisorption, while with physical adsorption one would expect attractive forces in some cases due to the adsorbed molecules, and a cooperative effect would be observed.

[9] A. R. Miller, *The Adsorption of Gases on Solids* (Cambridge Monographs on Physics, 1949).

relationship between experiment and theory than can exist in many adsorption studies.

As has been suggested, these two examples of adsorption studies on filaments may be regarded as first approximations to investigations on known surfaces. These are by no means the only experiments on single wires, but they are among the better known. Another type of surface that can be regarded as a first approximation to a uniform surface is the surface of an evaporated metal film. Probably the best known workers in this field are O. Beeck[10,11] and his associates. A brief summary of his work will be presented.

Beeck's investigations on metal films were carried out in a deliberate attempt to study the action of gases on simple surfaces. Indeed, part of his work was an effort to examine adsorption on surfaces of known crystal orientation. If nickel is evaporated in a high vacuum onto glass, the resulting film shows no preferred crystal orientation, but if the nickel is evaporated in nitrogen at a pressure of 1.0 mm Hg, electron diffraction studies show that the film is oriented so that the plane parallel to the backing is a (110) crystal plane. Beeck reported adsorption isotherms of the simple Langmuir type on such a nickel film with ethylene, carbon monoxide, nitrogen, hydrogen, and oxygen as adsorbates at −183°C and 23°C. All of the films were stable and irreversible and of the chemisorbed type, except nitrogen at −183°C which was reversible and of the van der Waals type.

The amount of each of the gases adsorbed at saturation was compared with the saturation amount of carbon monoxide adsorbed, and the following table was constructed.

| gas   | CO | $H_2$ | $N_2$ | $C_2H_4$ | $O_2$ |
|-------|----|-------|-------|----------|-------|
| ratio | 1  | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | 2 |

Thus compared with carbon monoxide, whose molecules are supposed to occupy one adsorption site, hydrogen and nitrogen molecules require two sites; ethylene molecules require four sites; and oxygen adsorbs with two molecules per site. Hydrogen is pictured by Beeck as dissociated,

nitrogen as lying on the surface, and ethylene as being adsorbed by its four hydrogen atoms.

The saturation amounts of both carbon monoxide and hydrogen were observed to be nearly the same at −183°C and at 23°C, and the isotherms are very similar. The total of carbon monoxide adsorbed at 23°C was about 80 percent of that adsorbed at −183°C, while with hydrogen the saturation amounts were the same. Thus the total number of available sites did not vary markedly with temperature, indicating a uniform adsorbing surface.

Beeck observed that the hydrogen isotherm at 23°C has a gradual slope so that the amount adsorbed at $10^{-4}$ mm Hg was 80 percent of the amount adsorbed at $10^{-1}$ mm Hg. The added 20 percent was easily removed and therefore possessed a lower heat of adsorption. Beeck interpreted this result in terms of the picture presented by Roberts, according to which a completely adsorbed layer of molecules occupying two adjacent sites will be left with gaps or holes. This theory would account for a 16 percent increase in adsorption, the additional adsorption being essentially molecular.

The above results are often taken as examples of adsorption on a crystal face of known orientation. However, Beeck reports that the films were porous, a film with an apparent area of 30 cm² had an effective area of 10,000 cm². Beeck drew these conclusions both from the amount of gas adsorbed in a monolayer and from the fact that the amount of gas adsorbed increased linearly with the thickness of the film. Because the electron diffraction studies yield information only about the face parallel to the backing, the surfaces on the pores are unknown with respect to crystal orientation, and these surfaces account for an overwhelming percentage of the effective area. These considerations make questionable any conclusions about adsorption on a particular crystal face, but the general information about the nature of adsorption on a comparatively clean metal surface is significant in any event.

Beeck and his co-workers continued their extensive investigation of adsorption on evaporated metal films, and recently Beeck[11] has summarized this research. Using an adsorption calorimeter, Beeck studied the heat of adsorption of hydrogen on films of nickel, iron, and

[10] Beeck, Smith, and Wheeler, Proc. Roy. Soc. (London) **177A**, 62–89 (1940), esp. pp. 78–80.
[11] O. Beeck, *Advances in Catalysis* II (Academic Press, Inc., New York, 1950), pp. 151–194.

tungsten. On nickel at 23°C the heat of adsorption decreased uniformly with the amount of surface covered from 30,000 cal/mole to 15,000 cal/mole, the same curve being obtained for oriented and unoriented films. Very similar results were obtained at −183°C, heat of adsorption falling off more rapidly than at 23°C and reaching about 10,000 cal/mole when the monolayer was complete. On iron at 23°C similar results were found, but on iron at −183°C Beeck reports that the heat of adsorption was nearly constant at about 27,000 cal/mole until the surface was covered, when the heat of adsorption dropped abruptly to 10,000 cal/mole. These results Beeck interprets in terms of the mobility of the adsorbed atoms by means of the following argument. No surface, even the face of a single crystal, is completely uniform, due to impurities on the surface or in the solid near the surface, to cracks and ridges, or to the presence of previously adsorbed atoms; therefore, the heat of adsorption will vary over the surface of the solid. If the gas molecules strike the surface and are held fast, then as the surface is covered, there will be little probability that regions of high heat of adsorption will acquire gas before regions with low heat of adsorption. Each additional amount adsorbed will be over all kinds of sites, and the measured heat of adsorption will be the average heat of adsorption. When the surface is nearly covered, every vacant site will be affected by the neighboring adsorbed atoms in the manner suggested by Roberts, and the heat of adsorption will drop rapidly. If, on the other hand, the adsorbed molecules are mobile, upon adsorption each molecule or atom will at once move to the regions of highest heat of adsorption, and as the surface is covered, the heat of adsorption will gradually decrease. Thus on nickel at both 23°C and −183°C he pictures the atoms as immobile.

The measurements of Roberts and Beeck are of great interest not only because of the experimental skill involved, but also because the measurements were made on clean, relatively uniform metal surfaces. It is interesting to observe that these scientists do not agree on the interpretation of results with respect to mobility, for the type of data that Roberts interprets as characteristic of immobility, Beeck interprets as characteristic of mobility. This disagreement serves to emphasize the need of a clear understanding of the adsorption process from a microscopic point of view. While it is undoubtedly true that both Roberts and Beeck made measurements on relatively pure metals, and Roberts especially had a nearly gas-free surface, it is important to realize that neither of them worked with a surface known in microscopic detail.

## ADSORPTION ON SINGLE CRYSTALS

We may consider as a second approximation to an ideally known solid surface the exposed face of a single crystal. This discussion will be closed with a consideration of studies performed on such a surface. The difficulties in obtaining suitable surfaces for any adsorption study are great, but for single crystals they are even greater. It is not simple to obtain single crystals of material, particularly for metals, on which the adsorption studies to be considered were made. The size of metal single crystals is limited, which means that the obtainable surface area is of the order of square centimeters and the amount of adsorbed gas is extremely small. Furthermore, gas is adsorbed on all the exposed surfaces, and again the problem of obtaining all the exposed faces of a single crystal parallel to a given set of crystal planes is appreciable. Despite these difficulties, a number of experiments have been performed with single crystals. Under this category of experiments we will discuss the use of electron diffraction in the study of the structure of the adsorbed gas on the surface.

There have been several attempts to study physical adsorption on surfaces of known crystal orientation. Many of these have been on various salt crystals, where it was assumed that the small crystal grains used were single crystals and that each crystal exposed only one type of crystal face. As it is difficult to check this assumption, it is open to question. More significant results have recently been obtained by Rhodin[12,13] using single crystals of copper cut into thin slices, with the exposed face of known orientation. Rhodin determined adsorption isotherms and heat of adsorption curves with nitrogen at liquid air temperatures on the (100), (110), and (111)

[12] T. N. Rhodin, J. Am. Chem. Soc. **72**, 4343–4348 (1950).
[13] T. N. Rhodin, J. Am. Chem. Soc. **72**, 5691–5699 (1950).

faces of copper, by means of a strong but sensitive microbalance. While there is some doubt as a result of electron diffraction studies to be discussed later,[21] as to whether his surfaces can be regarded as gas free, Rhodin did observe slightly different characteristics for the different crystal faces, showing specificity for physical adsorption with respect to crystal structure. The heat of adsorption as a function of surface coverage showed a maximum at the completion of a monolayer, a result Rhodin interpreted in terms of the cooperative interaction of the adsorbed molecules.[8]

Rhodin met the problem of several exposed crystal faces by cutting the crystals into very thin slices. This means that the edges of these sheets are of unknown crystal structure, but the greatest part of the area is of known crystal structure. The small unknown part of the area is less important in physical adsorption than in chemisorption, since chemisorption in general can be expected to be highly specific for the adsorbent, while physical adsorption depends largely on the characteristics of the gas. It would therefore be desirable if a method could be found by which the amount of gas adsorbed only on that part of the surface whose structure is known could be measured. One possible method is the use of a radioactive tracer.

Preliminary measurements of the amount of carbon dioxide adsorbed on the (111) face of a nickel single crystal have been made by H. E. Farnsworth and the writer[14,15] using $C^{14}$ as a tracer. The procedure is briefly as follows. The crystal was outgassed by heating in high vacuum and the $CO_2$ was admitted at pressures of about $10^{-2}$ to $10^{-1}$ mm of Hg and at temperatures varying from room temperature to 200°C. After several minutes, the unadsorbed $CO_2$ was withdrawn, and the crystal was removed from the vacuum chamber and placed before a Geiger counter. As only irreversibly adsorbed gas is detected, the measurements were limited to chemisorption. By using a metal diaphragm to mask the edges of the crystal, only the gas on the known portions of the crystal surface was

counted, and by covering different parts of the surface the distribution of adsorbed gas could be determined. The results of these measurements showed that the small amounts of gas adsorbed on the metal area of about one square centimeter could be detected in this manner, but the results were not reproducible for apparently similar conditions, the amount of gas adsorbed varying by several hundred percent between readings. Fundamentally, the difficulty seems to stem from the fact that the crystal was in the air of the room during measurement, and that the air interacted with the surface in an unknown manner so that one could never be sure the surface conditions were reproduced. Order of magnitude calculations showed that the surface of nickel was probably never more than 15 percent covered. There is a possibility that the surface was poisoned by some gas such as oxygen which was only partially removed by outgassing. Calculations show that a 1 or 2 percent change in the amount of poisoning would be sufficient to account for variations of the order of a hundred percent in some of the observations. R. A. Beebe[16] has suggested that possibly the $CO_2$ was adsorbed only on previously adsorbed oxygen, the amount of oxygen varying in unknown amounts. These experiments are admittedly tentative in nature, but they indicate that the small amount of adsorbed gas involved can be measured on a restricted portion of an exposed area. Further work using radioactive isotopes would therefore seem to be in order.

The adsorption of gases on single crystals has been studied by means of electron diffraction. Low speed electrons probably penetrate only a few atomic layers and the resulting diffraction beams yield information about the arrangement of surface atoms. As these data can only be obtained under high vacuum conditions, if gas atoms on the surface of a solid are to be observed, they must be irreversibly adsorbed, or desorption must proceed fairly slowly after the gas has been removed. A great deal of information can be gained, however, about the behavior of adsorbed gases by this method.

The first to observe the presence of adsorbed gas with electron diffraction were Davisson and

[14] Albert D. Crowell and H. E. Farnsworth, Phys. Rev. 78, 351 (1950).

[15] Albert D. Crowell and H. E. Farnsworth, J. Chem. Phys. 19, 1206–1207 (1951).

[16] R. A. Beebe (private communication).

Germer.[17,18] In the investigation of electron-diffraction beams, they found that some gas was adsorbed in a monomolecular layer on the (111) face of a nickel single crystal. The gas was residual from the air and was unidentified, but there seemed no doubt that gas was present. The gas was in the form of a two-dimensional lattice, identical with the (111) face of nickel, but double-spaced. As the temperature was raised past 150°C the beams due to the gas vanished,[18] although there was no evidence that the gas had left the crystal. This result was interpreted as meaning that the two-dimensional crystal of gas had melted, forming a mobile monolayer. This result was of considerable interest to those interested in the mobility aspect of the adsorption problem.

The study of adsorbed gases by the use of electron diffraction was extended by H. E. Farnsworth.[19,20] Investigating the (100) face of copper and silver single crystals, Farnsworth reported that hydrogen at room temperature formed a double-spaced, face-centered lattice in a thin layer with pressures of hydrogen less than 0.5 mm Hg, and a thicker, single-spaced, simple cubic lattice with pressures of hydrogen of the order of 2 mm Hg on copper, and a weak double-spaced, face-centered lattice on silver. It is interesting to note that on copper measurements were made in a high vacuum after the hydrogen was pumped off. The second thick layer disappeared in a few minutes, but the thin, double-spaced, face-centered layer was extremely stable and remained after the copper had been heated to red heat in a vacuum. Farnsworth was able to conclude that the diffraction beams observed were due to surface layers, and not to gas molecules which penetrated the metal lattice. The gas was much more difficult to remove in the case of copper than in the case of either silver or nickel. The observation of the two phases or structures of adsorbed gas make it tempting to interpret these results in terms of physical and chemisorption occurring on the face of the crystal, since the thick layer which is easily removed

seems characteristic of physical adsorption, while the thin layer, difficult to remove, is probably monomolecular, and is specific with regard to the adsorbent—all properties associated with chemisorption. Regardless of whether this interpretation is wholly correct, Farnsworth's experiments throw considerable light on the problem of the structure of adsorbed gas on metals.

The use of electron diffraction in adsorption studies has been continued by Schlier and Farnsworth.[21] Nitrogen, air, and carbon dioxide were studied on the (100) face of copper, and a stable, single-spaced simple cubic structure forming at a pressure of the order of $10^{-4}$ mm Hg was found. These films were stable and complete monolayers were formed in ten minutes or less. These experiments also clearly show the extreme difficulty of obtaining a gas-free surface, as a double-spaced, face-centered structure persisted even after many hours of outgassing at red heat in high vacua.

## CONCLUSIONS

The preceding discussion, though brief, indicates some of the problems involved in the study of the adsorption of gases on solids. Particular stress has been laid on the problem of investigating adsorption on uniform surfaces, as these measurements seem closest to the theories of adsorption, many of which assume the simplest type of surfaces. The examples chosen of adsorption studies on metal wires, films and single crystals are by no means exhaustive, but they seem to illustrate the techniques used and the type of conclusions that have been drawn.

The first conclusion that one can draw from an historical study of this type is that the problem is far from finished. Despite the careful and brilliant work of Langmuir, Roberts, and Beeck, the surfaces studied by these men were still of an unknown nature with respect to crystal structure, cracks, and other surface irregularities. Experiments such as those performed by Rhodin are all too few, and the initial attempts with radioactive tracers have yielded very little in the way of adsorption data. The results of elec-

[17] C. Davisson and L. H. Germer, Phys. Rev. **30**, 705–740 (1927).

[18] L. H. Germer, Z. Physik **54**, 408–421 (1929), esp. p. 418.

[19] H. E. Farnsworth, Phys. Rev. **35**, 1131–1133 (1930).

[20] H. E. Farnsworth, Phys. Rev. **40**, 684–712 (1932).

[21] Robert E. Schlier and H. E. Farnsworth, Phys. Rev. **78**, 316 (1950).

tron diffraction show nothing about the amount of gas adsorbed or the magnitudes of the energies involved. All in all, there is a great deal to be learned.

The second conclusion that one may draw is that the problem is a difficult one. It is far from easy to prepare a surface with known crystal structure which is free from contamination by impurities and which can be outgassed. It is even difficult to know when a satisfactory or known surface has been obtained. Such a surface is small and the amount of gas it can adsorb is small, making the detection of amounts and energies exacting measurements requiring sensitive techniques.

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# Graduate Study and Research in Physics

## (Report of the Committee on Facilities for Graduate Study and Research in Physics)

A survey of staff available, space and library facilities, and research budgets of physics departments in American universities is summarized. Fields of study in institutions offering the Ph.D. and M.S. degrees are itemized.

A SURVEY of the facilities for graduate study and research in physics in American universities was undertaken in the fall of 1950 by a committee of the American Association of Physics Teachers. This survey endeavored to obtain the following information:

1. Number of staff members competent and available to guide graduate students in their research. (Part-time staff were counted proportional to their available time.)
2. Number of graduate students.
3. Number of graduate students engaged in research directed towards the Ph.D. degree.
4. Floor space for research. (Space for shops, storage, and library were not included.)
5. Budget for research. (Included supplies, equipment and expense and salaries of technicians, machinists, and research assistants, whether provided by the department or by contracts with industrial or government agencies. The salaries of permanent staff or temporary advisory research staff were not included.)
6. Library budget, both central and departmental, for physics books and periodicals.
7. Fields of study in which students may engage in research for the Ph.D. degree.

Replies were received during the fall semester of 1950 from most of the institutions contacted. This report includes the data from 67 institutions offering the Ph.D. degree[1] and from 52 offering only the master's degree. Of the 67 "Ph.D." institutions, 11 preferred not to report their research budget or left the item blank. Nine institutions left the item for floor space blank and 13 gave no library budget. The budget figures are not very reliable as some institutions have included research institutes with their departmental budget while others have not. For example, the physics department of the University of California at Berkeley has not included the Radiation Laboratory in its budget although many of the graduate students do their research there. Most institutions appear to have included in their budgets funds received from government and industrial grants. From the reports of comparative institutions it is possible to estimate the budgets and space of the 11 nonreporting institutions. In this way a total budget and area for research has been calculated. An arbitrary division has also been made into the larger and smaller institution by grouping those with 10 or fewer staff members in one group and those with 11 or more staff members in another group. The library budgets have been averaged by institutions, since the cost of periodicals and

[1] In a report by M. W. White, "Enrollments and Degrees Awarded to Physics Majors," Am. J. Phys. 19, 27 (1951), 68 institutions are listed as giving the Ph.D. degree. The following institutions are among the 68 reported by White, but are missing from this report: Case Institute of Technology, Catholic University, George Washington University, Radcliffe College, Rice Institute, St. Louis University, and University of Southern California.

TABLE I. Facilities for study and research in physics in institutions offering the Ph.D. degree (Fall semester, 1950).

| Institution | Staff | Graduate students | Research students | Fields of study |
|---|---|---|---|---|
| Boston University | 10 | 129 | 27 | c i l m o |
| Brown University | 11 | 44 | 23 | a c g j k m o |
| Buffalo, University of | 6 | 30 | 6 | b j k l m |
| California Inst. of Tech. | 27 | 65 | 54 | b c d e g i j k l m |
| California, University of (Berkeley) | 21 | 235 | 115 | b c e f g h i j k l m |
| California, University of (Los Angeles) | 16 | 125 | 35 | a b f g j l m |
| Carnegie Inst. of Tech. | 19 | 60 | 50 | c d g j k m n |
| Chicago, University of | 22 | 130 | 60 | b d g j k m n o |
| Cincinnati, University of | 8 | 32 | 6 | e m o |
| Colorado, University of | 10 | 43 | 8 | b c h i j l m |
| Columbia University | 20 | 234 | 95 | c d e f g h j k l m o |
| Connecticut, University of | 7 | 12 | 0 | a b c f i j m |
| Cornell University | 21 | 92 | 60 | b c d e f i j k l m n |
| Duke University | 8 | 37 | 25 | b c f g h i j l m |
| Florida, University of | 7.5 | 18 | 4 | g j k l m o |
| Fordham University | 5 | 50 | 10 | b c j o |
| Harvard University | 26.5 | 173 | 71 | a b c d e f h j k l m n o |
| Illinois Inst. of Tech. | 6 | 30 | 12 | c g l |
| Illinois, University of | 29 | 172 | 54 | b d g j k l m |
| Indiana University | 10 | 55 | 25 | b g j m o |
| Iowa State College | 25 | 70 | 40 | b c d e g i j k l m n o |
| Iowa, State University of | 7 | 40 | 15 | b f g h j k m |
| Johns Hopkins University | 13 | 75 | 25 | b c f g h i j k l m n o |
| Kansas State College | 5 | 27 | 2 | c f j k l n |
| Kentucky, University of | 7 | 21 | 3 | c g h i j l |
| Lehigh University | 12 | 26 | 12 | c f k m |
| Louisiana State University | 9 | 31 | 6 | b c d e g j k l m n o |
| Maryland, University of | 7 | 200 | 10 | o |
| Massachusetts Inst. of Tech. | 40 | 200 | 100 | a b c d e f g h i j k l m n |
| Michigan State College | 9 | 36 | 9 | a f h i j l m n o |
| Michigan, University of | 22 | 142 | 36 | a b c f g i j k l m |
| Minnesota, University of | 12 | 81 | 23 | c g m n o |
| Missouri, University of | 6 | 30 | 7 | a b c i k m n |
| Nebraska, University of | 5 | 19 | 4 | g k m |
| New Mexico, University of | 3 | 26 | 3 | b c m o |
| New York University | 10 | 210 | 32 | b c f h i k l m o |
| North Carolina State College (Raleigh) | 9 | 30 | 5 | i j k n o |
| North Carolina, Univ. of (Chapel Hill) | 8 | 28 | 15 | b c j k l m o |
| Northwestern University | 12 | 38 | 15 | c d g h i k l m n o |
| Notre Dame, University of | 10 | 50 | 17 | c g h j k m o |
| Ohio State University | 22 | 169 | 60 | b d f g h j k l |
| Oklahoma, University of | 9 | 35 | 14 | b e m |
| Oregon State College | 11 | 25 | 8 | c g h i l m n o |
| Oregon, University of | 6 | 22 | 4 | c f i j k l n o |
| Pennsylvania, University of | 16 | 120 | 60 | d k m n o |
| Pennsylvania State College | 8 | 64 | 24 | a c d f i j k l m n |
| Pittsburgh, University of | 10 | 40 | 13 | c l m o |
| Polytechnic Institute of Brooklyn | 6.5 | 113 | 9 | f k l m n |
| Princeton University | 23 | 42 | 16 | b g j k l m o |
| Purdue University | 22 | 94 | 38 | b d f g j k l m n |
| Rensselaer Polytechnic Inst. | 10 | 44 | 24 | c f j k m o |
| Rochester, University of | 15 | 50 | 47 | b g j m |
| Rochester, University of (Inst. of Optics) | 5 | 20 | 10 | i k l |
| Rutgers University | 8 | 25 | 10 | d e f h k m |
| Stanford University | 14 | 44 | 20 | c f g h k m n |
| Syracuse University | 9 | 52 | 16 | a b c i j k l m o |
| Tennessee, University of | 7 | 95 | 13 | c k l m o |
| Texas A & M, College of | 5 | 12 | 2 | a c j l n o |
| Texas, University of | 12 | 105 | 25 | a c e f g i j l m n o |
| Utah, University of | 8 | 35 | 15 | c f i j k l m n |
| Vanderbilt University | 9 | 48 | 18 | c i j k l m n o |
| Virginia, University of | 5 | 40 | 20 | b c g i j k o |
| Washington, State College of | 6 | 25 | 8 | b c d m n o |
| Washington University (St. Louis) | 9 | 53 | 47 | b g h j |
| Washington, University of | 12 | 100 | 26 | a b c e f g i k l m n |
| Wisconsin, University of | 16 | 106 | 70 | c d g j k l m n |
| Yale University | 19 | 75 | 40 | b c d e f g j k l m |

books is not a pronounced function of the number of users.

Table I gives the data for each reporting institution granting the Ph.D. degree regarding the staff, graduate students, and fields of research. Table II gives similar data for those institutions granting only the master's degree. Table III provides a summary of the material in Tables I and II and also includes information regarding the budget, space, and library provisions.

The apparent adequacy of both financial support for research and of faculty for the guidance of students may be somewhat misleading. A few institutions, with million-dollar budgets for nu-

TABLE II. Facilities for study and research in physics in institutions offering only the master's degree. (Fall semester, 1950).

| Institution | Staff | Graduate students | Fields of study |
|---|---|---|---|
| Alabama, University of | 4 | 12 | |
| Alabama Polytechnic Inst. | 5 | 7 | c e i j m o |
| Arizona, University of | 3 | 8 | |
| Arkansas, University of | 4 | 10 | a b c j l m n |
| Baylor University | 3 | 0 | a c |
| Boston College | 5 | 33 | a c l m |
| Bradley University | 3 | 5 | c f k m |
| Central Michigan College of Education | 2 | 6 | c i j |
| Claremont Graduate School | 3 | 1 | |
| Clark University | 2 | 0 | |
| Colorado A & M College | 5 | 8 | a b c l n |
| Delaware, University of | 4 | 15 | c f m |
| Denver, University of | 6 | 8 | b i l |
| De Paul University | 3 | 15 | c j o |
| Detroit, University of | 4 | 16 | a c g j k l m n |
| Emory University | 5 | 12 | |
| Georgetown University | 6 | 6 | a i k |
| Georgia Institute of Technology | 4 | 6 | c d f j |
| Georgia, University of | 4 | 5 | c f i j l |
| Hawaii, University of | 3 | 3 | |
| Idaho, University of | 4 | 10 | c i j l m |
| Kansas State Teachers College | 3 | 3 | b c h l m |
| Kent State University | 4 | 5 | a c i l n |
| Louisville, University of | 6 | 10 | |
| Maine, University of | 5 | 3 | |
| Mississippi State College | 1 | 0 | |
| Mississippi, University of | 4 | 4 | |
| Montana State Col. (Bozeman) | 4 | 3 | a c j o |
| Montana State Univ. (Missoula) | 3 | 2 | |
| Newark College of Engineering | 5 | 0 | |
| New Hampshire, University of | 8 | 10 | g i j k m o |
| North Dakota, University of | 2 | 2 | |
| Ohio University (Athens) | 4 | 6 | b c i j l n |
| Oklahoma A & M College | 7 | 14 | b f i |
| Redlands, University of | 2 | 1 | |
| Rhode Island State College | 4 | 5 | a c l n o |
| Smith College | 4 | 7 | b c l m |
| Stevens Institute of Technology | 3 | 50 | |
| Texas Christian University | 3 | 3 | a c f i m n |
| Tufts College | 7 | 12 | o |
| Tulane University | 8 | 14 | c i j l m n |
| Wayne University | 8 | 10 | |
| Western Reserve University | 4 | 15 | c i |
| Wyoming, University of | 6 | 7 | a c h i j k m o |

| | |
|---|---|
| a Acoustics | h Nuclear Moments |
| b Cosmic Rays | i Optics |
| c Electronics | j Radioactivity |
| d Low Temperature | k Solid State |
| e Magnetism | l Spectroscopy |
| f Microwaves | m Theoretical |
| g Nuclear Accelerators | n X-Rays |

o Other fields including: Astrophysics, Biophysics, Gas Discharges, Geophysics, Heat and Thermometry, Hydrodynamics, Mass Spectroscopy, Molecular Beams, Molecular Structure, Polymers, and Upper Atmosphere.

| | |
|---|---|
| a Acoustics | h Nuclear Moments |
| b Cosmic Rays | i Optics |
| c Electronics | j Radioactivity |
| d Low Temperature | k Solid State |
| e Magnetism | l Spectroscopy |
| f Microwaves | m Theoretical |
| g Nuclear Accelerators | n X-Rays |

o Other fields including: Astrophysics, Biophysics, Gas Discharges, Geophysics, Heat and Thermometry, Hydrodynamics, Mass Spectroscopy, Molecular Beams, Molecular Structure, Polymers, and Upper Atmosphere.

TABLE III. Facilities for graduate study and research. Summary of statistics for the Fall semester 1950.

| | Institutions offering the Ph.D. degree | | | Institutions offering only the master's degree | Total for all institutions |
|---|---|---|---|---|---|
| | Small | Large | All | | |
| Number of institutions | 39 | 28 | 67 | 44 | 111 |
| Staff competent to guide research ($S$) | 298 | 531 | 829 | 188 | 1,017 |
| Graduate students ($G$) | 1,924 | 2,892 | 4,816 | 366 | 5,182 |
| Research students ($R$) | 504 | 1,278 | 1,782 | | |
| Total research workers ($R+S$) | 802 | 1,809 | 2,611 | | |
| Annual research budget ($\$$) | 2,560,000 | 9,730,000 | 12,290,000 | 359,000 | 12,649,000 |
| Research space (ft²) | 346,000 | 840,000 | 1,186,000 | 111,000 | 1,296,000 |
| Graduate students per staff ($G/S$) | 6.4 | 5.4 | 4.8 | 2.0 | |
| Research students per staff ($R/S$) | 1.7 | 2.4 | 2.1 | | |
| Budget per research worker ($\$$) | 3,250 | 5,400 | 4,700 | | |
| Space per research worker (ft²) | 430 | 460 | 455 | | |
| Books and periodicals per institution ($\$$) | 1,370 | 1,850 | 1,550 | 515 | |

clear and related research, appreciably raise the average budget. There are not only wide fluctuations in the average number of research students per staff member for different institutions, but within an institution many faculty members have no research students while others accept and attempt the guidance of as many as twenty students working for the Ph.D. degree.

It should be noted that the research budget includes a proportionate share of the salaries of machinists and technicians. The data obtained in this survey indicates that the more productive Institutions provide a research budget of about $5000 for each research worker and about 500 ft² of space. The library budget for these institutions is about $2000 a year for books and periodicals.

*Committee on Facilities for Graduate Study and Research in Physics*

R. B. Brode, *Chairman*,
Joseph B. Platt,
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# Hall Effect and Ponderomotive Force in Simple Metals*

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(Received August 16, 1951)

The Bloch-Sommerfeld theory of conductivity is applied to the calculation of momentum lost by conduction electrons because of collisions with the crystal lattice. The momentum lost by the conduction electrons, suitably averaged, is interpreted as a force per unit volume acting on the crystal lattice. The calculations provide the basis for a microscopic interpretation of the ponderomotive force $\mathbf{f} = (1/c)(\mathbf{j} \times \mathbf{H})$ per unit volume.

In the case of metals which conduct in a single band, the ponderomotive force $\mathbf{f}$ is produced entirely by the action of the Hall electric field on the crystal lattice. When conduction takes place in more than one band $\mathbf{f}$ is partially caused by the action of the Hall electric field on the crystal lattice and partially caused by collisions of conduction electrons with the lattice.

## I. INTRODUCTION

WHEN an electrical conductor carrying a current is placed in a transverse magnetic field as illustrated in Fig. 1, two effects are observed. Firstly, the conductor experiences a force $\mathbf{f}$ per unit volume, which, as indicated by $f_y$ in Fig. 1, is in a direction perpendicular to the directions of current and magnetic field. This force is called the ponderomotive or the technical magnetic force. Secondly, an emf may be observed in the same direction as the force. This emf and its associated electric field inside the conductor constitute the familiar Hall effect. In the calculations that follow it will be apparent

that there is an intimate relation between the ponderomotive force and the Hall effect.

If an external electric field is applied to a metal, quasi-free electrons in unfilled bands are accelerated. Collisions of electrons with the metal lattice prevent the electrons from accelerating indefinitely. During collisions, the lattice exerts a force on the electrons which changes their momenta, and a corresponding reaction force must act on the lattice. Forces on the ion lattice may also be produced by electric fields. The net force from these two agencies constitutes a macroscopic body force acting on the conductor.

In this paper, the net force on the ion lattice will be calculated for the case of constant applied electric field $E_x$ and for the case of constant applied electric and magnetic fields $E_x$ and $H_z$. A general outline of the calculations will be given and details will be omitted as far as possible in specific cases.

## II. CALCULATION PROCEDURE

The calculations follow a more or less standard treatment of high temperature conductivity such as may be found in Mott and Jones' book.[1]

In the absence of applied fields $F_0(\mathbf{k})d\mathbf{k}$ is the number of electrons in states whose wave vectors lie in the range $(\mathbf{k}, d\mathbf{k})$, per unit volume of the metal, where

$$F_0(\mathbf{k}) = \frac{2}{8\pi^3} \frac{1}{1 + \exp[\epsilon(\mathbf{k}) - \epsilon_0]/k_b T}, \quad (1)$$

in which $\epsilon(\mathbf{k}) =$ electron energy as a function of wave vector $\mathbf{k}$, $\epsilon_0 =$ Fermi energy, $k_b =$ Boltzmann constant, and $T =$ absolute temperature; $d\mathbf{k}$ means the volume element in $\mathbf{k}$-space $dk_x dk_y dk_z$ and the range $(\mathbf{k}, d\mathbf{k})$ means the range between $k_x$ and $k_x + dk_x$, $k_y$ and $k_y + dk_y$, $k_z$ and $k_z + dk_z$.

In the presence of applied fields the new equilibrium distribution function $F(\mathbf{k}, \mathbf{r})$ satisfies the Boltzmann equation,

$$\frac{\partial F}{\partial t}\Big)_{\mathrm{drift}} + \frac{\partial F}{\partial t}\Big)_{\mathrm{collisions}} = 0. \quad (2)$$

Calculations will be restricted to isothermal conditions and constant fields so that $F$ is independent of position $\mathbf{r}$ to good approximation.

[1] N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Clarendon Press, Oxford, 1936).
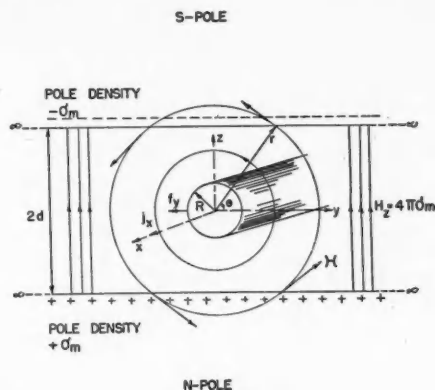


FIG. 1. An electrical conductor carrying a current, placed between the poles of a magnet whose pole faces are infinite in the $x$ and $y$ directions in order to produce the uniform field $H_s$. The field $\mathcal{K}$ of the current acts on the pole distribution on the pole faces to produce a force equal and opposite to the force acting on the conductor.

That is, $F(\mathbf{k}, \mathbf{r}) \cong F(\mathbf{k})$ only. In this case $\partial F/\partial t)_{\mathrm{drift}}$, the rate of change of the distribution function resulting from the acceleration of electrons by applied fields, may be expressed as

$$\partial F/\partial t)_{\mathrm{drift}} \cong -1/h(\mathbf{K} \cdot \nabla_k)F, \quad (3)$$

where

$$(\mathbf{K} \cdot \nabla_k) = K_x \frac{\partial}{\partial k_x} + K_y \frac{\partial}{\partial k_y} + K_z \frac{\partial}{\partial k_z},$$

and $\mathbf{K} =$ force on an electron because of applied fields.[2]

If $W(\mathbf{k}, \mathbf{k}')d\mathbf{k}'$ is the probability per unit time that an electron in a state in $(\mathbf{k}, d\mathbf{k})$, as a result of a collision with the lattice, makes a transition to a new state $\mathbf{k}'$ lying in $(\mathbf{k}', d\mathbf{k}')$ then the rate of change of the distribution function because of collisions may be expressed as

$$\frac{\partial F}{\partial t}\Big)_{\mathrm{collisions}} = \int W(\mathbf{k}, \mathbf{k}')[F(\mathbf{k}') - F(\mathbf{k})]d\mathbf{k}', \quad (4)$$

the integration being taken over all final states.

Above the Debye characteristic temperature of the lattice a relaxation time $\tau$ may be introduced such that

$$\partial F/\partial t)_{\mathrm{collisions}} = -(F - F_0)/\tau(k). \quad (5)$$

[2] See, for example, H. Jones and C. Zener, Proc. Roy. Soc. (London) **A144**, 101 (1934).

Collisions of electrons with the lattice are approximately elastic, so that

$$W(\mathbf{k}, \mathbf{k}') \cong W(k, \theta)\delta(k - k'), \qquad (6)$$

where $\theta =$ the angle between $\mathbf{k}$ and $\mathbf{k}'$, $\delta(k - k') = 0$ if $k \neq k'$, and $\delta(k - k') = 1$ if $k = k'$. Then from Eqs. (2), (4), (5), and (6)

$$\frac{1}{\tau(k)} = \int W(k, \theta) \left\{ 1 - \frac{\partial F(\mathbf{k}')}{\partial t} \right)_{\text{drift}} \bigg/$$
$$\frac{\partial F(\mathbf{k})}{\partial t} \bigg)_{\text{drift}} \right\} dS', \quad (7)$$

in which $dS'$ is an element of the surface formed by the ends of all final state $\mathbf{k}'$ vectors.

### (a) Calculation of Electrical Currents

When the applied fields have been specified, Eq. (2) is solved for the distribution function $F(\mathbf{k})$. Clearly, $F(\mathbf{k})d\mathbf{k}$ is the number of electrons per unit volume of metal, which occupy states in the wave-vector range $(\mathbf{k}, d\mathbf{k})$. An electron in a state $\mathbf{k}$ produces a current as though it had a velocity given by $\mathbf{v}_k = (1/h)\nabla_k \epsilon(\mathbf{k})$ so that the electrons in the wave-vector range $(\mathbf{k}, d\mathbf{k})$ produce a current density $-e\mathbf{v}_k F(\mathbf{k})d\mathbf{k}$. The total current density $\mathbf{j}$ from all states corresponding to $+\infty > k_x > -\infty$, $+\infty > k_y > -\infty$, and $+\infty > k_z > -\infty$ is found by integration over these limits. That is,

$$\mathbf{j} = -e \int \mathbf{v}_k F(\mathbf{k})d\mathbf{k}. \qquad (8)$$

### (b) Calculation of Forces Acting on the Lattice

When an electron is scattered by lattice perturbations from a state $\mathbf{k}$ of momentum $h\mathbf{k}$ to a new state $\mathbf{k}'$ of momentum $h\mathbf{k}'$ there is a change of momentum of the electron of amount $h(\mathbf{k}' - \mathbf{k})$. The effect of the lattice on the electron is to change its momentum by $h(\mathbf{k}' - \mathbf{k})$, and there must be an equal and opposite reaction on the lattice. If one computes the total change of momentum in unit time of all the electrons which make transitions, the reaction on the lattice must be a force numerically equal and opposite in sign to this change in momentum of the electrons. The calculation is made for a unit volume of the metal.

In unit time, of the $F(\mathbf{k})d\mathbf{k}$ electrons originally in $(\mathbf{k}, d\mathbf{k})$, $F(\mathbf{k})d\mathbf{k}W(\mathbf{k}\mathbf{k}')[1 - F(\mathbf{k}')]d\mathbf{k}'$ electrons make transitions to some of the $[1 - F(\mathbf{k}')]d\mathbf{k}'$ unoccupied states in $(\mathbf{k}', d\mathbf{k}')$. Each transition involves a momentum change $h(\mathbf{k}' - \mathbf{k})$ of the assembly of electrons. The total change $\Delta \mathbf{P}$ of momentum in unit time of the assembly of electrons is found by integrating over all initial and final states. Thus

$$\Delta \mathbf{P} = \int h(\mathbf{k}' - \mathbf{k})F(\mathbf{k})W(\mathbf{k}, \mathbf{k}')$$
$$\times [1 - F(\mathbf{k}')]d\mathbf{k}d\mathbf{k}'. \quad (9)$$

The equal and opposite reaction on the lattice is $\mathbf{f}_P = -\Delta \mathbf{P}$, which is the force per unit volume. Making use of Eq. (6) we have

$$\mathbf{f}_P = h \int (\mathbf{k} - \mathbf{k}')F(\mathbf{k})W(k, \theta)$$
$$\times [1 - F(\mathbf{k}')]d\mathbf{k}d\mathbf{S}'. \quad (10)$$

If the ion lattice has an average charge $q$ per unit volume there will also be a force

$$\mathbf{f}_E = q\mathbf{E}, \qquad (11)$$

because of the action of the applied electric field $\mathbf{E}$. The total force acting on unit volume of the metal will be

$$\mathbf{f} = \mathbf{f}_P + \mathbf{f}_E. \qquad (12)$$

### (c) Solutions of the Boltzmann Equation

A solution is anticipated of the form $F(\mathbf{k}) = F_0(k) + \chi(\mathbf{k})(\partial F_0/\partial \epsilon)$ where $\chi(\mathbf{k})$ is so small that products such as $\chi E$, $(\partial \chi/\partial \epsilon)E$ may be neglected. The function $F(\mathbf{k})$ has been obtained for an applied electric field $E_x$ and for electric and magnetic fields $E_x$ and $H_z$ with the following simple representations of the band structure of metals:

*Metal* A-1—a single-band electronic conductor in which one band is nearly empty and contains $N_s$ electrons per unit volume. The energy levels are given by the relation,

$$\epsilon_s(\mathbf{k}) = h^2 k^2/2m_s. \qquad (13)$$

*Metal* A-2—a single-band hole conductor in which one band is nearly filled except for $N_d$ unoccupied states or holes per unit volume. Near the top of the band

$$\epsilon_d(\mathbf{l}) = \epsilon_m - h^2 l^2/2m_d. \qquad (14)$$

*Metal* B—a two-band conductor in which one band is nearly filled ($N_d$ holes per unit volume) and one band is nearly empty ($N_s$ electrons per unit volume). Electron energies in the two bands are given by Eqs. (13) and (14).

## III. RESULTS OF CALCULATIONS AND PHYSICAL INTERPRETATIONS

### (a) Applied Electric Field Only

($\mathbf{K} = -eE_x\mathbf{i}_1$, where $\mathbf{i}_1$ is a unit vector in the $x$-direction.) The appropriate solution $F(\mathbf{k})$ of the Boltzmann equation was employed to evaluate the current integral, Eq. (8), and the force integral, Eq. (10). Details of the calculations for this case are given in Appendix I.

*Metal* A–1—(electronic conduction). For this case the current integral is

$$j_x = (N_s e^2/m_s)\tau_s(k_0)E_x, \qquad (8.1)$$

where $k_0$ is the wave-vector magnitude corresponding to the Fermi energy $\epsilon_0$. The force integral is

$$f_{Px} = \frac{-m_s}{e\tau_s(k_0)}j_x = -N_s eE_x. \qquad (10.1)$$

The total body force on the lattice must also include a term $f_{Ex}$ because of the action of the electric field $E_x$. Since there are $N_s$ conduction electrons per unit volume, the lattice has a charge $+N_s e$ per unit volume and

$$f_{Ex} = +N_s eE_x. \qquad (11.1)$$

The total force acting on unit volume of the lattice is

$$f_x = f_{Px} + f_{Ex} = 0. \qquad (12.1)$$

In Fig. 2 the directions of the current and corresponding net momentum of the electrons are illustrated for this case. The result that $f_x = 0$ is, of course, to be expected, since the metal is electrically neutral. However, the details of the process which produce this result in a conductor are perhaps not trivial.

*Metal* A–2—(hole conduction). Here we have

$$j_x = \frac{N_d e^2}{m_d}\tau_d(l_0)E_x, \qquad (8.2)$$

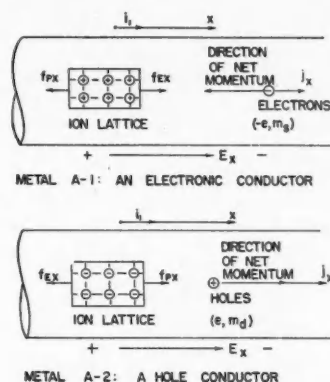$$f_{Px} = \frac{m_d}{e\tau_d(l_0)}j_x = +N_d eE_x. \qquad (10.2)$$



FIG. 2. Directions of various physical quantities involved in one-dimensional conduction and the mechanical equilibrium of the lattice. Note the relative directions of current density and net momentum of the conduction particles.

In this case, as illustrated in Fig. 2, the net momentum associated with the conduction electrons or holes[3] is in the opposite direction to that found for metal A–1, or in the same direction as the current. If there are $N_d$ conducting holes per unit volume, the lattice may be considered to have a charge $-N_d e$ per unit volume so that

$$f_{Ex} = -N_d eE_x. \qquad (11.2)$$

The total force on the lattice is given by

$$f_x = f_{Ex} + f_{Px} = 0. \qquad (12.2)$$

*Metal* B—(simultaneous hole and electron conduction). Solutions of the Boltzmann equation $F_s(\mathbf{k})$ and $F_d(\mathbf{l})$ are found appropriate to the $s$- and $d$-bands. Then

$$j_x = j_x(s) + j_x(d) = \frac{N_s e^2\tau_s(k_0)}{m_s}E_x$$
$$+ \frac{N_d e^2\tau_d(l_0)}{m_d}E_x \quad (8.3)$$

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[3] One may talk about conduction electrons of effective mass $-m_d$ and charge $-e$ or conduction holes of effective mass $+m_d$ and charge $+e$. For example, see the paper by W. Heisenberg, Ann. Physik **10**, 888 (1931), in which it is demonstrated that the Schrödinger equation for $n$ electrons in a band containing $N$ states may be transformed to a Schrödinger equation for $N-n$ holes which is similar except that the signs of the charge and effective mass of the electrons are reversed in the terms containing external electric and magnetic fields.

$$f_{Px} = f_{Px}(s) + f_{Px}(d) = \frac{-m_s}{e\tau_s(k_0)}j_x(s)$$
$$+ \frac{m_d}{e\tau_d(l_0)}j_x(d) \quad (10.3)$$
$$= (N_d - N_s)eE_x.$$

The lattice has a charge $(N_s - N_d)e$ per unit volume so that

$$f_{Ex} = (N_s - N_d)eE_x \quad (11.3)$$

and

$$f_x = f_{Ex} + f_{Px} = 0 \quad (12.3)$$

as before.

### (b) Applied Electric and Magnetic Fields

$(\mathbf{K} = -e\mathbf{E} - (e/c)(\mathbf{v}_k \times \mathbf{H})$, where $\mathbf{E} = \mathbf{i}_1 E_x + \mathbf{i}_2 E_y$ and $\mathbf{H} = \mathbf{i}_3 H_z$; $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ are unit vectors in the direction of the $x$, $y$, and $z$ axis, respectively).

*Metal* A–1—(electronic conduction). The two components of the current integral are

$$j_x = \sigma_s \left( E_x - \frac{\sigma_s H_z}{N_s ec} E_y \right)$$

and $(8.4)$

$$j_y = \sigma_s \left( E_y + \frac{\sigma_s H_z}{N_s ec} E_x \right) = 0,$$

where $\sigma_s = N_s e^2 \tau_s(k_0)/m_s$. The two components of the force integral are

$$f_{Px} = \frac{-m_s j_y}{e\tau_s(k_0)} = -N_s eE_x,$$

and $(10.4)$

$$f_{Py} = \frac{-m_s j_y}{e\tau_s(k_0)} = 0.$$

It has been assumed that $j_y = 0$ and the approximation of neglecting $(\sigma_s H_z/N_s ec)^2$ compared to unity has been justified. This approximation was employed in the calculation of expressions (8.4) and is usually good up to fields of $10^5$ gauss.

Taking the charge per unit volume of the lattice to be $+N_s e$ as before, it follows that

$$f_{Ex} = N_s eE_x$$

and $(11.4)$

$$f_{Ey} = N_s eE_y.$$

The total forces on the lattice are given by the equations

$$f_x = f_{Ex} + f_{Px} = 0,$$
$$f_y = f_{Ey} + f_{Py} = N_s eE_y. \quad (12.4)$$

The symbol $E_y$ represents the well-known Hall electric field which, from Eq. (8.4), has the value

$$E_y = -\frac{1}{N_s ec}j_x H_z. \quad (15.4)$$

From Eqs. (12.4) and (15.4) it follows that

$$f_y = (-1/c)j_x H_z, \quad (16.4)$$

where $f_y$ is the familiar ponderomotive force.

For metal A–1, the ponderomotive force is produced entirely by the action of the Hall electric field on the crystal lattice. This result can be understood simply in the following superficial description: Under the influence of the applied field $E_x$, electrons acquire a mean drift velocity $\bar{v}_x = -j_x/N_s e$. When the magnetic field $H_z$ is applied electrons tend to drift in the $y$-direction under the influence of the mean Lorentz force $(+e/c)\bar{v}_x H_z$. However the constraint $j_y = 0$ produces the Hall electric field $E_y$ such that the mean force on an electron in the $y$-direction is zero; that is, $-eE_y + (e/c)\bar{v}_x H_z = 0$, so that $E_y = (1/c)\bar{v}_x H_z = -(1/N_s ec)j_x H_z$. When equilibrium is attained there are no net external forces acting on the conduction electrons in the $y$-direction and hence no net $y$-component of momentum. Therefore, no net $y$-component of momentum can be communicated to the lattice by collisions of conduction electrons with the lattice and the force $f_y$ must be entirely caused by the action of the Hall electric field on the charge $+N_s e$ per unit volume of the lattice. Then clearly

$$f_y = N_s eE_y = -(1/c)j_x H_z.$$

*Metal* A–2—(hole conduction). Here we have

$$j_x = \sigma_d \left( E_x + \frac{\sigma_d H_z}{N_d ec} E_y \right),$$

and $(8.5)$

$$j_y = \sigma_d \left( E_y - \frac{\sigma_d H_z}{N_d ec} E_x \right) = 0,$$

where

$$\sigma_d = N_d e^2 \tau_d(l_0)/m_d.$$

For

$$j_y = 0 \quad \text{and} \quad (\sigma_d H_z/N_d ec)^2 \ll 1,$$
$$f_{Px} = N_d eE_x,$$

and $(10.5)$

$$f_{Py} = 0.$$

Taking the charge per unit volume of the lattice to be $-N_d e$, it is clear that

$$f_{Ex} = -N_d e E_x$$

and    (11.5)

$$f_{Ey} = -N_d e E_y.$$

In addition,

$$f_x = f_{Ex} + f_{Px} = 0$$

and    (12.5)

$$f_y = f_{Ey} + f_{Py} = -N_d e E_y.$$

In this case the Hall electric field is positive; from Eqs. (8.5) it is found that

$$E_y = +(1/N_d ec) j_x H_z$$    (15.5)

and

$$f_y = -(1/c) j_x H_z.$$    (16.5)

The ponderomotive force $f_y$ is produced entirely by the action of the Hall electric field on the lattice in this case also.

*Metal* B—(simultaneous hole and electron conduction). In this case conduction takes place in both the $s$- and $d$-bands and most of the foregoing quantities have two components. For example,

$$j_x = j_x(s) + j_x(d),$$

and

$$j_y = j_y(s) + j_y(d),$$

where $j_x(s)$ and $j_y(s)$ are given by the right-hand sides of Eq. (8.4), and $j_x(d)$, $j_y(d)$ are given by the right-hand sides of Eq. (8.5). Combining these equations one obtains

$$j_x = \sigma(E_x - R\sigma H_z E_y),$$

and    (8.6)

$$j_y = \sigma(E_y + R\sigma H_z E_x),$$

where

$$\sigma = \sigma_s + \sigma_d,$$

and

$$R = \frac{1}{N_d N_s ec} \frac{N_d \sigma_s^2 - N_s \sigma_d^2}{(\sigma_s + \sigma_d)^2}.$$

Now, for the conditions $j_y = 0$ and $(R\sigma H_z)^2 \ll 1$, it follows that

$$f_{Px} = -\frac{m_s j_x(s)}{e\tau_s(k_0)} + \frac{m_d j_x(d)}{e\tau_d(l_0)} = -(N_s - N_d)e E_x,$$

and    (10.6)

$$f_{Py} = -\frac{m_s j_y(s)}{e\tau_s(k_0)} + \frac{m_d j_y(d)}{e\tau_d(l_0)}$$

$$= -(N_s - N_d)e E_y - \frac{1}{c} j_x H_z.$$

Taking the charge per unit volume of the lattice to be $(N_s - N_d)e$, we find

$$f_{Ex} = (N_s - N_d)e E_x,$$

$$f_{Ey} = (N_s - N_d)e E_y,$$    (11.6)

$$f_x = f_{Ex} + f_{Px} = 0,$$

$$f_y = f_{Ey} + f_{Py} = -(1/c) j_x H_z,$$    (12.6)

as in the previous cases. In this case the ponderomotive force $f_y$ is partially caused by momentum transfer and partially caused by the Hall electric field. The quantities $f_{Py}$ and $f_{Ey}$ may also be expressed as follows:

$$f_{Ey} = \left(\frac{1}{N_d} - \frac{1}{N_s}\right)\left(\frac{N_d\sigma_s^2 - N_s\sigma_d^2}{(\sigma_s + \sigma_d)^2}\right)\left(\frac{-j_x H_z}{c}\right),$$

$$f_{Py} = \left(\frac{\sigma_s}{N_s} + \frac{\sigma_d}{N_d}\right)\left(\frac{N_s\sigma_d + N_d\sigma_s}{(\sigma_s + \sigma_d)^2}\right)\left(\frac{-j_x H_z}{c}\right),$$    (16.6)

$$\frac{f_{Ey}}{f_{Py}} = (N_s - N_d)\frac{(N_d\sigma_s^2 - N_s\sigma_d^2)}{(N_d\sigma_s + N_s\sigma_d)^2}.$$

The particular case of $N_d = N_s$ is of interest because then $f_{Ey} = 0$, $f_{Py} = -(1/c) j_x H_z$, showing that the ponderomotive force is entirely caused by momentum transfer. This situation is realized in, for example, the element nickel.

## CONCLUSIONS

It is customary to discuss the ponderomotive force in terms of Maxwell's macroscopic theory. For example, if an electrical conductor carrying a current $I_x = \pi R^2 j_x$ is placed between the poles of a magnet in a uniform field $H_z$ as in Fig. 1, the magnetic field produced by the current acts on the surface distribution of poles on the pole faces to produce the force $F_y$ per unit length of magnet. The force is given by the equation

$$F_y = 2\int_{-\infty}^{\infty} \mathcal{K} \sin\theta \sigma_m dy,$$

where the integration is taken along the upper pole face. Coordinates are indicated in Fig. 1. Since $\mathcal{K} = 2I_z/cr$, $\sigma_m = H_z/4\pi$, $y = r\cos\theta$, and $d = r\sin\theta$, one obtains $F_y = (1/c)I_zH_z$. The symbol $F_y$ represents the reaction force acting on the magnet. The action force acting on the conductor per unit volume of conductor must be

$$f_y = -(1/c)j_zH_z.$$

The results obtained with Maxwell's theory are, of course, correct, but give little insight into the nature of the process which produces the action force in the conductor.

The present calculations, based on the Bloch-Sommerfeld theory of conductivity, describe the process in the conductor and give explicit results whose derivation illuminates the details of the phenomenon.

An intimate relation between the Hall effect and the ponderomotive force becomes apparent. (Such a relation can not be visualized from Maxwell's theory since Maxwell's theory contains no reference to the Hall effect.) For single-band conductors the ponderomotive force is produced entirely by the action of the Hall electric field on the crystal lattice. When conduction takes place in more than one band, it is partially because of the Hall electric field and partially because of momentum lost by the conduction electrons as a result of collisions with the lattice.

The author would like to express his appreciation to Professors R. Becker, J. E. Goldman, and E. M. Pugh of Carnegie Institute of Technology, for helpful discussions and criticism.

## APPENDIX

### I. Calculation of the Current and Force Integrals

The calculation is made for the case $\mathbf{K} = -eE_z\mathbf{i}_1$ and $\epsilon_s(\mathbf{k}) = (\hbar^2/2m_s)k^2$. The Boltzmann equation for this case is, from Eqs. (2), (3), and (5),

$$\frac{-eE_z}{\hbar}\frac{\partial F}{\partial \epsilon}\frac{\partial \epsilon}{\partial k_z} + \frac{F - F_0}{\tau(k)} = 0. \qquad (A.1)$$

A solution is anticipated of the form $F = F_0 + (\chi\partial F_0/\partial\epsilon)$, where products such as $\chi E_z$, $(\partial\chi/\partial\epsilon)E_z$ may be neglected. Substituting this form

of $F$ in Eq. (A.1) we find that

$$\chi \cong \frac{e\tau(k)E_z}{\hbar}\frac{\partial\epsilon}{\partial k_z}. \qquad (A.2)$$

(a) *Calculation of the Current Integral.* From Eq. (8),

$$j_z = -e\int (\mathbf{v}_k)_z F(\mathbf{k})d\mathbf{k},$$

where

$$(\mathbf{v}_k)_z = 1/\hbar(\partial\epsilon_s/\partial k_z) = (\hbar/m_s)k_z,$$

and $F(\mathbf{k}) = F_0(k) + \chi(\partial F_0/\partial\epsilon)$. Since $k_z$ is antisymmetric, $F_0(k)$ is symmetric in $k_z$, and the integration is taken between $k_z = -\infty$ and $k_z = +\infty$, the part of the integral involving $(\mathbf{v}_k)_z F_0(k)$ vanishes leaving

$$j_z = -e\int (\mathbf{v}_k)_z\chi(\partial F_0/\partial\epsilon)d\mathbf{k}.$$

It is convenient to choose a volume element in $\mathbf{k}$-space between two surfaces of constant energy, $\epsilon$ on one surface and $\epsilon + d\epsilon$ on the neighboring surface. Then the volume element in $\mathbf{k}$-space is $d\epsilon dS/|\nabla_\mathbf{k}\epsilon|$, where $dS$ is an element of the surface of constant energy. Then $j_z$ can be expressed as

$$j_z = -e\int\left\{\int_0^\infty (\mathbf{v}_k)_z\chi(\partial F_0/\partial\epsilon)(d\epsilon/|\nabla_\mathbf{k}\epsilon|)\right\}dS.$$

The function $F_0$, given by Eq. (1), has the property that for $(k_bT/\epsilon_0)\ll 1$, $(\partial F_0/\partial\epsilon)\cong 0$ for all energies except in the neighborhood of the Fermi energy $\epsilon = \epsilon_0$. The foregoing condition is satisfied for all temperatures of interest so that the $\epsilon$-integration can easily be carried out by making use of the well-known theorem[1] for integrals involving Fermi functions, that

$$\int_0^\infty \psi(\epsilon)(\partial F_0/\partial\epsilon)d\epsilon \cong (-1/4\pi^3)\psi(\epsilon_0),$$

where $\psi(\epsilon)$ is an arbitrary function of $\epsilon$. Thus the current integral is simplified to an integral over the Fermi surface which is, in this case, the sphere $k_x^2 + k_y^2 + k_z^2 = 2m_s\epsilon_0/\hbar^2$ of radius $k_0 = (2m_s\epsilon_0/\hbar^2)^{\frac{1}{2}}$. The simplified current integral is

$$j_z = \frac{e}{4\pi^3}\int\frac{(\mathbf{v}_k)_z\chi dS}{|\nabla_\mathbf{k}\epsilon|}. \qquad (A.3)$$

By employing polar coordinates $(k, \phi, \theta)$, so that $k_x = k_0 \sin\theta \cos\phi$ and $dS = k_0^2 \sin\theta d\theta d\phi$, Eq. (A.3) is easily found to reduce to $j_x = (e^2\tau(k_0)E_x/m_s) \times (1/4\pi^3)(4\pi k_0^3/3)$. In the right-hand expression $(1/4\pi^3)$ is the density of states in $\mathbf{k}$-space and $(4\pi k_0^3/3)$ is the volume of the Fermi sphere which contains $N_s$ occupied states. Therefore the result for $j_x$ may be expressed as

$$j_x = \frac{N_s e^2 \tau(k_0)}{m_s} E_x. \qquad (A.4)$$

Since

$$\left.\frac{\partial F}{\partial t}\right]_{\text{drift}} \cong \frac{-eE_x}{\hbar} \frac{\partial F_0}{\partial \epsilon} \frac{\partial \epsilon}{\partial k_x},$$

Eq. (7) for $\tau(k)$ reduces in this particular case to

$$1/\tau(k) = \int W(k, \theta)(1 - k_x'/k_x)dS'. \quad (A.5)$$

Equation (A.5) may be placed in standard form by the usual transformation[1]

$$k_x = k \cos\alpha,$$
$$k_x' = k(\cos\alpha \cos\theta + \sin\alpha \sin\theta \cos\phi), \quad (A.6)$$
$$dS' = k^2 \sin\theta d\theta d\phi,$$

in which $k = |\mathbf{k}|$; $\alpha$ is the angle between $\mathbf{k}$ and the $k_x$ axis; $\theta$, the angle between $\mathbf{k}$ and $\mathbf{k}'$; $\phi$, the angle between the plane containing $\mathbf{k}$ and the $k_x$-axis and the plane containing $\mathbf{k}'$ and $\mathbf{k}$. The result is

$$1/\tau(k) = 2\pi k^2 \int_0^\pi W(k, \theta)(1-\cos\theta)\sin\theta d\theta. \quad (A.7)$$

(b) *Calculation of the Force Integral.* Equation (10) for this case has only the component

$$f_{Px} = \hbar \int F(\mathbf{k}) W(k, \theta)[1 - F(\mathbf{k}')](k_x - k_x')d\mathbf{k}dS'.$$

Consider first that

$$J(\mathbf{k}) = \hbar \int [1 - F(k')]W(k, \theta)(k_x - k_x')dS'.$$

Substituting $F(\mathbf{k}')$ and making the transforma-

tion (A.6), we find that

$$J(\mathbf{k}) = \frac{\hbar k_x}{\tau}[1 - F_0(k)] - \frac{e\tau E_x \hbar^2}{m_s} \frac{\partial F_0}{\partial \epsilon}\left[\frac{k_x^2}{\tau_1} - \frac{k^2 - k_x^2}{2\tau_2}\right],$$

where

$$1/\tau_1 = 2\pi k^2 \int_0^\pi W(k, \theta)(1-\cos\theta)\sin\theta \cos\theta d\theta$$

and $\qquad\qquad\qquad\qquad\qquad\qquad (A.8)$

$$1/\tau_2 = 2\pi k^2 \int_0^\pi W(k, \theta)\sin^3\theta d\theta.$$

Then

$$f_{Px} = \int F(\mathbf{k}) J(\mathbf{k})d\mathbf{k}$$

$$= \hbar \int k_x \frac{F_0(k)[1 - F_0(k)]}{\tau(k)}d\mathbf{k} + \frac{eE_x \hbar^2}{m_s}\int k_x^2 \frac{\partial F_0 d\mathbf{k}}{\partial \epsilon}$$

$$- \frac{eE_x \hbar^2}{m_s}\int \left[k_x^2 + k_x^2\frac{\tau}{\tau_1} - (k^2 - k_x^2)\frac{\tau}{2\tau_2}\right]F_0 \frac{\partial F_0}{\partial \epsilon}d\mathbf{k}$$

$$- \left[\frac{eE_x}{m_s}\right]^2 \hbar^3 \int k_x \left[\frac{k_x^2\tau^2}{\tau_1} - (k^2 - k_x^2)\frac{\tau^2}{2\tau_2}\right]\left(\frac{\partial F_0}{\partial \epsilon}\right)^2 d\mathbf{k}.$$

$$(A.9)$$

The first and last integrals vanish because the integrands are antisymmetric in $k_x$. Substituting Eqs. (A.8) for $\tau_1$ and $\tau_2$ and making use of the identity $\int k^2\phi(k)d\mathbf{k} = 3\int k_x^2\phi(k)d\mathbf{k}$, where $\phi(k)$ is any function of $k$, we find that the third integral is also zero. Hence

$$f_{Px} = \frac{eE_x \hbar^2}{m_s}\int k_x^2 \frac{\partial F_0}{\partial \epsilon}d\mathbf{k};$$

this is evaluated in the same way as the current integral to obtain

$$f_{Px} = -N_s eE_x$$
$$\qquad\qquad\qquad\qquad\qquad (A.10)$$
$$= -\frac{m_s}{e\tau(k_0)}j_x,$$

by Eq. (A.4).

Calculations for other cases discussed in this paper may involve somewhat more labor, but involve no essential differences in principle or procedure from the calculation described previously.

# Projects in Electrical Laboratory

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After performing nine standard experiments, the student chooses a project from a list of eleven for which the department has the necessary apparatus. Some of these projects require independent thinking and planning; others require only intelligent and selective use of printed references. Three men ordinarily work together, planning and executing their project in four weeks, with minimum help from the instructor. Each team hands in a written final report and may make an oral progress report to the class. Surprising enthusiasm is generated, and much is learned about the management of a small research project. A list of project titles appears in this paper.

E ACH section of our laboratory course in electricity for sophomore engineers meets for three hours per week. The first hour of each period is spent in class discussion. The course begins with the performance of about ten standard experiments chosen by the course chairman from a list of about fifteen for which printed or typed instructions are available. A sample list is given in Table I. The students work in pairs and do their own wiring.

During the last four weeks of the semester the student performs a project that he selects from a short list supplied by the two instructors in his section and chosen by them from a longer list such as the one given in Table II. Some of these projects require independent thought; others require only intelligent selection and use of printed references; but all require planning by the student.

Our laboratory instructors are graduate students who hold teaching fellowships. Early in the semester each instructor chooses a project that he is willing to supervise and one or two others that he is willing to sponsor. He inherits from last year's staff complete written instructions for the project that he is to supervise, and after performing this project himself, he issues a corrected and modified version of the instructions to those of his colleagues who have agreed to sponsor it. (None of this written material is shown to students.) At this time new projects may be proposed and adopted.

About two weeks before projects are started by the students, each section of 24 men meets with its two instructors. The titles of four, five, or six projects are announced, and the general plan for choosing and performing projects is explained. The students are also informed that teams of two, three, or four men will be organized. Thanks to the flexibility of this scheme, little trouble is encountered in organizing the whole section harmoniously. Typically, in a given section of 24 men there will be eight teams working on four different projects; thus each instructor will have charge of four teams and two projects. During the four weeks in which projects are performed a definite space in the laboratory is allotted to each project, and all the apparatus used by the various groups that have chosen this project is kept there for the entire time, if possible.

The student can usually be guided toward a wise choice of project. Good men tend to work together, to know what they want, and to choose one of the harder projects. In order to avoid misfits it is desirable to record the preferences of the poorer students first.

A week after projects are chosen, each half-section of twelve men may hold a meeting at

TABLE I. A typical list of standard experiments.

1. Measurement of the earth's magnetic field.
2. Ammeter check by electrolysis of water.
3. Wheatstone bridge.
4. Natural period and critical damping of wall galvanometer.
5. Potentiometer.
6. Kirchhoff's laws.
7. Sensitivity of a galvanometer and emf of a thermocouple.
8. Electromagnetic induction.
9. Reactance and resistance of an iron-core choke coil.
10. Static characteristics of a triode.

which a spokesman for each team tells the class what his team plans to do and what methods are to be used. The team members have compiled this information by discussion among themselves or from sources found by themselves or indicated by the instructor. The class is encouraged to quiz the spokesman until he makes his meaning clear and justifies what he says. In this way each student in the group may benefit by learning what is being done by other project teams. The instructor sits at the back of the room and says as little as possible. He acts as moderator and timekeeper.

Project time is divided between experimental work, library research, team conferences, and report writing as the students see fit. Before the last class period each team submits a verbal progress report and a rough draft of the final written report. The final report may be a collaboration in which references are cited and the individual contribution of each member of the team is indicated. The instructor is authorized to retain a copy for departmental files if he so announces in advance.

A surprising amount of enthusiasm is generated and much is learned about the management of a small research project. Techniques are critically reviewed and many ideas that were only dimly apprehended in the routine course work are illuminated and made clear. No doubt some students deprive themselves of these benefits by consulting old reports or those who wrote them; it is well to discourage and control such activities by discarding some projects and adding others.

It is hardly necessary to say that the graduate assistant and the course chairman also learn a great deal. If we could see how to circumvent the difficulties, we should like to extend the project method to our laboratory work in sound, light, and heat; but major construction problems may confront the student if he attempts a project in one of these subjects. It is evident that the project method is well adapted to electricity, for in this subject the student has only to select the proper pieces of equipment and to make the proper connections.

TABLE II. Some projects and some apparatus likely to be used in them.

1. **Survey of the earth's field inside and outside the laboratory.** Magnetometer, bar magnet, compass, dipping needle, earth inductor, ballistic galvanometer.
2. **Field between the polepieces of a strong permanent magnet.** Magnetron magnet, search coil made by student, ballistic galvanometer, fluxmeter, standard solenoid, resistance boxes.
3. **Measurement of several high resistances.** Unknowns ranging from one megohm to a thousand megohms, standard megohm, several standard capacitors, wall galvanometer, high resistance dial boxes, electrometer-tube dc amplifier.
4. **Measurement of several low resistances.** Several ammeter shunts as unknowns, several low resistance standards, millivoltmeter, ammeter, resistance boxes, rheostats, student potentiometer.
5. **Characteristics of photocells.** Vacuum phototube, gas phototube, Photronic cell, selenium cell, light sources and filters, dark room with photometric bench and standard lamp, motor-driven sector disks, neutral wedges, neon lamp, af oscillator, af amplifier, wall galvanometer, rectifier voltmeter.
6. **Common uses of the cathode-ray oscilloscope.** Rectifier voltmeter, vacuum-tube voltmeter, af oscillator, resistance boxes, rheostats, Variac, dial inductor box, dial capacitor box.
7. **Study of the forces between bar magnets.** Several pairs of magnets, chemical balance with weights, frame and other facilities for constructing a five-thread sling for suspending a bar magnet and constraining it to plane motion in a circular path without tilting.
8. **Exploration of the axial magnetic field of a short solenoid.** Ballistic galvanometer, ammeter, rheostat, resistance boxes, standard capacitor or standard solenoid, three test solenoids. The test solenoids have primaries alike but secondaries different; secondaries are concentrated in slots of different radii with turns adjusted to same mutual inductance with primaries; secondaries can slide inside primaries along a common axis.
9. **Resonance in a series circuit.** Two air-core inductors, Variac, adjustable paper capacitor, dial capacitor box, variable air capacitor, resistance boxes, rheostats, rectifier voltmeters, af oscillator, af amplifier, vacuum-tube voltmeter.
10. **Study of ac phase relations with the oscilloscope.** The same list as in 9, plus the cathode-ray oscilloscope.
11. **Calibration of a platinum resistance thermometer.** Callendar bridge, vacuum flask for liquid oxygen, jar of cracked ice, hypsometer for boiling water.

# NOTES AND DISCUSSION

## A Physical Model to Demonstrate Nuclear and Paramagnetic Resonance

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AS a result of recent discoveries in physics, new fields of research in electron and nuclear resonance have been opened. The models herein described were constructed to present a clear-cut picture of the physical concepts and effects associated with magnetic resonance.

We shall first discuss a model which demonstrates an effect that one might expect in nuclear resonance. For the purpose of a demonstration, we have chosen the hydrogen nucleus. Since the proton is believed to be a very small charged particle spinning about an axis through its center of mass, some of its properties should be similar to those of a small bar magnet. Now let the small bar magnet $N$-$S$ in Fig. 1 represent the proton. The proton is assumed to be in an externally applied magnetic field whose direction is indicated by the arrow $H$ in the diagram. If one is to assume that the bar magnet or proton is spinning about an axis connecting its north and south poles, then from the laws of dynamics one would expect that it could undergo precession about an axis parallel to the external field. By turning the crank shown in the diagram we can demonstrate the precession of a proton. The quantum-mechanical picture gives only two possible orientations for the proton; one is indicated in the diagram, and the other is obtained by interchanging the north and south poles, a reorientation that is accomplished by flipping the magnet over with two strings $c$ and $d$ fastened off-center under the bar magnet. In order to demonstrate what takes place when a photon is absorbed, we flip the proton from one orientation to the other while it is precessing. By using the relation

$$E = \mu H \cos\theta,$$

where $E$ is the energy, $\mu$ the magnetic moment, and $\theta$ the angle between the proton and the external field, one can easily show how much energy is absorbed or emitted in reorienting the proton.
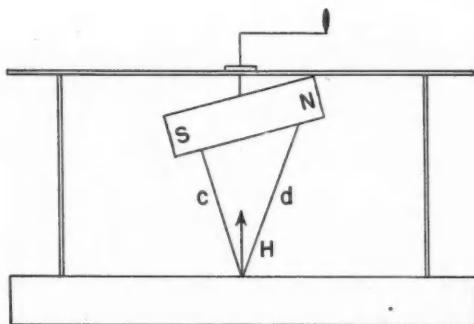
We shall next discuss a model shown in Fig. 2 which demonstrates some effects that one might expect in paramagnetic resonance. For example, consider the copper atom; it has one electron in its outermost shell, but when it combines with other elements to form crystals, such as $CuSO_4 \cdot 5H_2O$, it must give up two electrons. One of the electrons must come from its next outermost shell, and the vacancy which it leaves appears to behave very much like the presence of an electron, provided we assume that all other electrons around the atom partially neutralize the effective charge of the nucleus. Since the absence of an electron appears to behave so much like an electron, we assume that it is one and represent it by the small sphere in Fig. 2. The arrow attached to the sphere is to indicate the axis about which it is assumed to be spinning. The other particle in which we are interested is the nucleus, located in the middle of the gap between the pole pieces of the model. Rather than two orientations like those of the
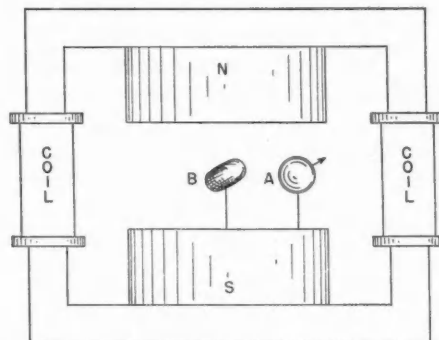


FIG. 2. Model for paramagnetic resonance: $A$, the electron with arrow indicating its axis of spin; $B$, oblate spheroidal nucleus; $N$ and $S$ represent the north and south poles of the electromagnet, respectively.

proton, we let this nucleus have four possible orientations in an external magnetic field corresponding to four different energy states for the nucleus. Since one normally thinks of an electron as moving in an orbit about the nucleus, the model was constructed so that the electron could be moved in a circle about the nucleus as center. For the external magnetic fields of the magnitude in which we are interested, the coupling between the electron and nucleus is broken down and each becomes quantized in the external field. This quantization is demonstrated by having the nucleus in one of its four possible orientations and the electron is one of its two possible orientations. As a crank under the pole pieces is turned, causing the electron and nucleus to revolve like a merry-go-round, the precessions are indicated by the motion of the arrow on the electron and the changing appearance of the oblate spheroidal nucleus, the precession of the electron about the axis parallel to the external field being the faster.



FIG. 1. Magnetic analog of proton: the bar magnet $NS$ represents the proton, which precesses about the magnetic field $H$.

In order to demonstrate what happens when a photon is absorbed, we use the same concept as for the proton except in this case it is the electron that acts like a small bar magnet, changing its orientation in the external magnetic field by going from its lowest to its highest spin state. The magnetic field at the position of the electron is caused by the field produced by the electromagnet plus the field produced by the nucleus. By the use of strings and rubber bands in the hollow rod supporting the electron, the electron spin is changed from one orientation to the other while both the electron and nucleus are precessing. For the four possible orientations of the nucleus, there are four different values of the magnetic field at the position of the electron, and because there are nuclei in all four energy states in the copper sulfate crystal, one would obtain four spectral lines in the microwave region.

The orbital angular momentum of the electron, which is quite small for paramagnetic resonance, will not have the same value for all energy states. Difficulties in building the model prevented us from showing this small change in the orbital motion as the electron goes from one state to the next. We should add that in order to obtain additional information about the atom there are other small effects that must be considered when precision measurements are made.

With these models the following phenomena can be demonstrated:

(1) Orientation of the axis of spin for a proton, electron, or nucleus relative to an external magnetic field giving rise to different energy states: (a) Two possible orientations for the proton, (b) two possible orientations for the electron, and (c) four possible orientations for the nucleus.

(2) Larmor precession about an axis parallel to the external magnetic field for each of the two possible orientations for the proton and electron and each of the four possible orientations for the nucleus.

(3) Orbital motion of an electron. (The electron moves in a circle about an axis parallel to the external field with the nucleus at the center.)

(4) Changes in energy states giving rise to absorption, while the particles are undergoing Larmor precession: (a) Proton changes from its lowest to its highest spin states in the presence of an externally applied magnetic field. (b) Electron changes from its lowest to its highest spin state in the presence of an externally applied magnetic field plus the field caused by the nucleus in any one of its four possible orientations.

(5) Changes in energy states giving rise to emission which is demonstrated in the same manner as absorption except the proton and electron go from the highest to the lowest spin state.

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## Historic Demonstrations

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IN the planning of demonstration experiments a conflict is common which may be illustrated by the conventional demonstration of x-rays. The instructor is tempted
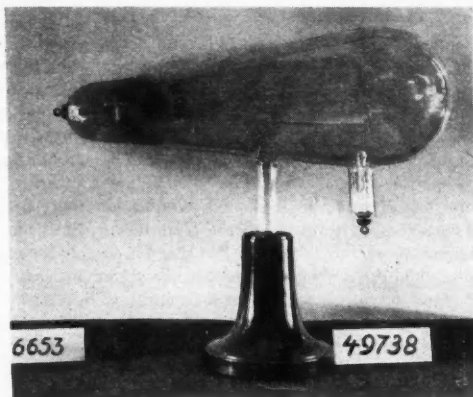


FIG. 1. Röntgen's discharge tube (courtesy Deutsches Museum, Munich).

to show modern equipment which gives intense effects visible over the entire lecture room. On the other hand, modern x-ray equipment comes completely encased in a sealed box with only a power supply cord to be plugged into a 110-volt outlet. The contents of the box must be accepted by the student on authority. In order to show to the student what is essential in the experiment and, at the same time, give him historical perspective, the writer went to the opposite extreme and duplicated the simple experiment by which Röntgen, in 1895, discovered the rays he called x-rays.

In the Deutsches Museum in Munich, the historical exhibit of science and technology, it is not possible to identify the particular tube with which Röntgen made his discovery.[1] However, the Deutsches Museum kindly supplied photographs of various tubes used by Röntgen. One of these photographs, included here as Fig. 1, shows a conventional discharge tube. It is 10 inches long and is easily duplicated. The pressure must be sufficiently low for the discharge, when operated by a good-sized induction coil with a hammer interrupter, to produce intense fluorescence of the glass. Röntgen's induction coil was 27 inches long. When the discharge tube is completely wrapped in black paper and the discharge turned on, one sees in the darkened room light on a fluorescent screen which is placed near the glass bulb opposite the cathode. A piece of metal inserted between the discharge tube and the screen cuts off the effect. By reversing the current it can be shown that the fluorescence appears only for one direction of the current. When the fluorescent screen is placed close to the glass bulb wrapped in black paper, the fluorescence is bright enough to be seen throughout an ordinary physics lecture room.

Röntgen's discharge tube (Fig. 1) resembles a tube which happens to be available in many laboratories where it is used for demonstrating that a metal cross placed in the path of cathode rays casts a shadow. When the cross, which is hinged, is turned down, this apparatus is a good duplicate of Röntgen's tube and shows the effect aforementioned.

This tube, as well as Röntgen's tube, is presumably built of soft glass, while our duplication of Röntgen's tube is built of Pyrex. The material makes no noticeable difference.

Another great discovery that can easily be demonstrated in its original form is that of radioactivity by Becquerel (1896). Becquerel placed a piece of pitchblende on a charged plate holder whose shutter was closed. The pitchblende was left there for about two days. After development the plate turned out to be blackened. When a key was placed between the pitchblende and the plate holder, its shadow was seen on the negative. Here we are not concerned with the gradual development of Becquerel's argument who, at the start, actually searched for a penetrating fluorescence excited by sunlight in uranium salts. He soon discovered that his results were independent of the sunlight. This interesting development of the discovery can well be traced in Becquerel's original papers.[2]

[1] The discovery is described in the book of Otto Glasser, *W. C. Röntgen* (Charles C. Thomas, Springfield, Ill., 1934), Chapters 3 and 4.
[2] H. Becquerel, Compt. rend. **122**, 420, 501, 559 etc. (1896).

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## Error in the Vibrating Chain Problem

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I READ with a great deal of interest John Satterly's article "Some Experiments in Dynamics, Chiefly on Vibrations" in a recent issue[1] of this journal. I was particularly interested in the experiment on the "Up and Down Vibrations of a Hanging Chain Partly Counterbalanced by a Suspended Body," which I used in a course in advanced dynamics, and the experimental results agreed very closely with Professor Satterly's published results. However, I question his theoretical solution for the downward movement of the chain.
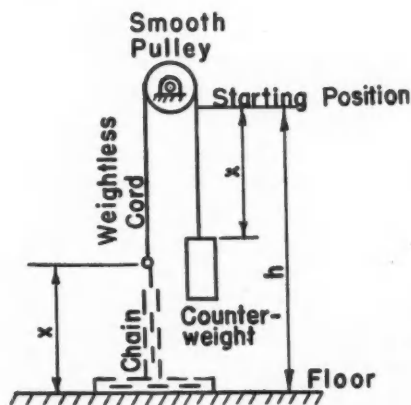


FIG. 1. A counterweight pulls a flexible chain from a heap by means of a cord passing over a pulley. The diagram shows the situation a short time after the beginning of the first upward motion of the chain. The counterweight never hits the floor, nor does the whole length of chain ever leave the floor. Hence the motion is oscillatory.

A restatement of the problem is as follows: "A fine uniform chain is collected in a heap on a horizontal table, and to one end is attached a fine string which passes over a smooth pulley (of negligible mass) vertically above the chain and carries a weight equal to the weight of a length $a$ of the chain. Prove that the length of the chain raised before the weight comes to rest is $a\sqrt{3}$ (assumed that all of the chain does not leave the table) and find the length suspended when the weight next comes to rest."

Figure 1 shows the chain displaced upward a distance $x$, and Fig. 2 shows the free body diagrams of the chain and counterweight during the upward motion of the chain. Let $L$=total length of chain, $m$=mass per unit length of chain, and $v=dx/dt$ = velocity of moving chain and counterweight. From Fig. 2(b)
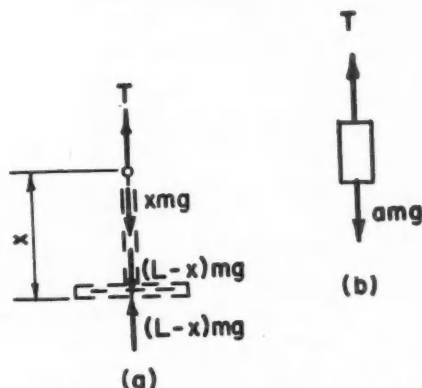
$$amg - T = am(dv/dt). \tag{1}$$



FIG. 2. Free body diagrams of (a) the chain in upward motion, (b) the counterweight in motion.

From Fig. 2(a)

$$T - xmg = d(xmv)/dt = mxdv/dt + mv^2. \tag{2}$$

The elimination of $T$ from Eqs. (1) and (2) yields

$$m(a-x)g = m(a+x)dv/dt + mv^2. \tag{3}$$

This is the differential equation for the upward motion of the chain; it agrees with Professor Satterly's. The solution of this differential equation gives $a\sqrt{3}$ for the position of zero velocity.

For the downward motion Professor Satterly gives the equation of motion as

$$m(a-x)g = m(a+x)(dv/dt) - mv^2, \tag{4}$$

and the solution of this equation gives $a/\sqrt{3}$ for the next position at which the velocity is zero.

I contend that this is not the correct differential equation for the downward motion of the chain. During the downward movement of the chain the table must act with an additional force sufficient to destroy the momentum of the links of the chain hitting the table, this additional force can readily be shown to be $mv^2$. The free body diagram of the chain for this portion of its motion is shown in Fig. 3.

Hence

$$T - xmg + mv^2 = d(xmv)/dt = mx(dv/dt) + mv^2. \quad (5)$$

The elimination of $T$ from Eqs. (5) and (1) gives

$$m(a-x)g = m(a+x)(dv/dt). \quad (6)$$

This is the correct equation of motion when the chain is moving downward.

Equation (6) can readily be developed from the motion of the mass center of the entire chain.

Let $\bar{x}$ = position of the mass center of the entire chain.

$$\bar{x} = (xm)(x/2)/Lm = x^2/2L$$
$$d\bar{x}/dt = xv/L \text{ and } d^2\bar{x}/dt^2 = (x/L)(dv/dt) + v^2/L.$$

Using the equation $\Sigma F_x = M d^2\bar{x}/dt^2$ and the free body diagram of Fig. 3 there results

$$T - xmg + mv^2 = Lm\{(x/L)(dv/dt) + v^2/L\}. \quad (7)$$

Again upon the elimination of $T$ from Eqs. (1) and (7) there results

$$m(a-x)g = m(a+x)(dv/dt),$$

which is the same as Eq. (6).

Very interesting results are also obtained when the problem is approached from an energy viewpoint.

Let $T$ = kinetic energy of the system, $U$ = potential energy of the system, and $E = T + U$ = total energy. Therefore,

$$T = \tfrac{1}{2}m(a+x)v^2,$$

and

$$U = mgx^2/2 + amg(h-x),$$

when the table is used as a reference plane for the potential energy.

The time rate of change of energy is

$$(dE/dt) = m(a+x)v(dv/dt) + \tfrac{1}{2}mv^3 + mgxv - amgv$$
$$= v[m(a+x)(dv/dt) + \tfrac{1}{2}mv^2 + mg(x-a)]. \quad (8)$$

By making use of Eq. (3) the above equation becomes

$$(dE/dt) = -\tfrac{1}{2}mv^3.$$

This is the rate of loss of energy on the upward motion. For the downward motion Eq. (6) is used and Eq. (8) becomes

$$(dE/dt) = \tfrac{1}{2}mv^3.$$

At first glance this looks inconsistent, but $v$ is negative during the downward motion, hence the rate of loss of energy for the same $v$ is identical for either upward or downward motion. However, if Professor Satterly's Eq. (4) is used, my Eq. (8) becomes

$$(dE/dt) = \tfrac{3}{2}mv^3.$$

This does not seem reasonable to me. I am not sure, but I feel that the rate of loss in energy on the way up should be the same as the rate of loss on the way down for the same velocity. If this is true my Eq. (6) is correct and not Professor Satterly's Eq. (4).

The solution of Eq. (6) is

$$v^2 = 2g[-x + 2a \ln(a+x)] + c,$$

and using the condition that $x = a\sqrt{3}$ when $v = 0$, it becomes

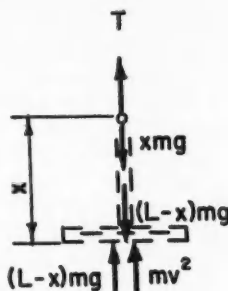$$v^2 = 2g[-x + 2a \ln(a+x)] - 2g[-a\sqrt{3} + 2a \ln(1+\sqrt{3})a]. \quad (9)$$



FIG. 3. Free body diagram of the chain in downward motion.

When this equation is solved for the value of $x$ that makes $v = 0$ it is found that

$$x = 0.412a.$$

When the value of $a = 88.6$ cm is used, the experimental and theoretical results of Professor Satterly and my theoretical values are compared as follows:

| | Exper. | Theor. (Satterly) | Theor. (Davis) |
|---|---|---|---|
| Chain rose to | 148.8 cm | $a\sqrt{3} = 153.3$ cm | $a\sqrt{3} = 153.3$ cm |
| Chain fell to | 50.4 cm | $a/\sqrt{3} = 51.1$ cm | $0.412a = 36.5$ cm |
| Chain rose to | 119.0 cm | $1.369a = 121.2$ cm | $1.488a = 131.8$ cm |

The experimental value of 50.4 cm checks the value of 51.1 cm much closer than my 36.5 cm. However, the 51.1 cm and my value 36.5 cm were determined by assuming that the chain rose to 153.3 cm but actually the chain only rose to 148.8 cm. Now I assume that the reason the chain did not rise to 153.3 cm is because of energy lost as a result of friction and swaying of the chain. On the downward motion there would be additional friction losses and the chain would not reach the value of 36.5 cm but come to rest at a higher value, obviously at 50.4 cm. If the 51.1 cm is correct, how could the chain come to rest at 50.4 cm since the friction would hinder its motion, not help it, and cause it to stop well above the 51.1 cm?

The great difference between my value of 36.5 cm, and the experimental value of 50.4 cm may partially be accounted for on examination of the potential energy of the system at its extreme positions where the energy is entirely potential. That is

$$U = \tfrac{1}{2}mgx^2 + amg(h-x)$$

and

$$dU = mgxdx - amgdx.$$

Hence

$$dx = dU/mg(x-a).$$

Now consider the effect of a loss of potential energy $dU$ at the two extreme positions of $a\sqrt{3}$ and $0.412a$. That is

$$dx_1 = dU/mg(a\sqrt{3} - a) = 1.36dU/mga,$$

and

$$dx_2 = dU/mg(0.412a - a) = -1.70dU/mga;$$

therefore

$$dx_2 = -1.25dx_1.$$

Hence for the same loss in potential energy the change in $x$ in the lower position would have to be 1.25 times as great as it would be in the higher position. Furthermore, as $x$ approaches $a$ the ratio of $dx_2$ to $dx_1$ becomes greater. If $x = a/\sqrt{3}$, then

$$dx_2 = -1.74 dx_1.$$

Therefore, for the same energy loss the change in the value of $x$ at the position $a/\sqrt{3}$ would be considerably greater than it would be for the position $0.412a$. For this reason I believe that the experimental results tend to confirm the value of 36.5 cm rather than the value 51.1 cm.

In addition to the above evidence, I would like to call attention to the 8th edition of Routh's *Dynamics* Part I, Article 300, page 247, Example 1, where a textbook problem[2] similar to this is worked out, and for the portion of the problem comparable to this one, an equation is given that agrees with mine and not the one used by Professor Satterly.

[1] J. Satterly, Am. J. Phys. **18**, 405–416 (1950), especially p. 414.
[2] Routh, *Dynamics* (Macmillan Company, New York), eighth edition, Part I, p. 247.

# LETTERS TO THE EDITOR

## Transmission Zone Plates

THE recent article,[1] "Studies of Transmission Zone Plates," brings out additional interesting facts about zone plates and reminds me of some experimental plates and tests made at the University of Florida in 1940. It was an interesting exercise to show, mathematically, that the plates could be used to form virtual images, or that $1/b = 1/p - 1/q$, for paraxial rays. Experimental tests showed that the plates had essentially the properties of a thin lens in regard to virtual image formation and magnification. Intensity of the light for these observations had to be rather large.

Some thought was given to possible applications of x-ray "pin-hole" cameras using zone plates to increase the intensity of the x-ray images on photographic films. Consideration of the problem of getting proper intensity attenuation in the radiation beam across the zone plate indicated that a plate had little probable value. In addition, the difficulty of producing small-diameter plates with a sufficient number of opaque zones appeared to make the use of the single aperture, or central zone, the practical solution.

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[1] Ora E. Myers, Jr., Am. J. Phys. **19**, 359–65 (1951).

## A Note on Dieterici's Reduced Equation of State

ALTHOUGH it is now fashionable to represent $(p, v, T)$ relations by a virial expansion, Dieterici's equation, $p(v-b) = RT \exp(-a/RTv)$, nevertheless retains a measure of popularity and is regarded by Beattie and Stockmayer[1] as one of the three classical equations of state. Possibly this is because the equation has a theoretical basis; because it is in some respects superior to van der Waals' equation (to which it reduces when $a$ and $b$ are small); and because, as indicated by its originator,[2] the values of its three parameters are readily obtained, by the usual procedure, in terms of the critical constants $p_c$, $v_c$ and $T_c$. It is, in consequence, to be regretted that a recent search of the likely textbooks in four London libraries has shown that the reduced equation of state to which these values lead is more often than not misquoted as $p_r(2v_r - 1) = T_r \exp 2(1 - v_r^{-1})$. In the hope that the common and fairly obvious omission, possibly originating in a printer's error, may not be perpetuated, it seems worth while to point out that the correct equation is

$$p_r(2v_r - 1) = T_r \exp 2(1 - v_r^{-1} T_r^{-1}).$$

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[1] *Reports on Progress in Physics* (The Physical Society, London, 1940). Vol. 7, p. 203.
[2] Wied. Ann. **69**, 685 (1899).

## A Mercury Light Source

AN economical mercury light source has been most recently described by Hilton.[1] A more convenient source may be obtained directly from a G.E. type RS Reflector Sunlamp which is available at the nearest drugstore for about $8.50.

This source has the advantage of being a self-contained unit which will operate satisfactorily in an ordinary lamp socket connected to the nearest 110-v ac line. It has the appearance of a photoflood lamp and contains a built in reflector; this eliminates radiation from all except the front of the tube, so that the only required additional equipment to the lamp is a shallow box with a suitable opening in the bottom.

A certain amount of continuous spectrum is present because of the glowing internal ballast, but for sophomore work this presents no impediment. It is anticipated that when the ballast becomes defective it will be possible to open the envelope, supply an external ballast, and proceed in the manner described by Hilton. The mercury-arc element is supposedly a separately sealed unit.

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[1] Wallace A. Hilton, Am. J. Phys. **19**, 248 (1951).

### Pressure within a Bubble

THE two-bubble paradox, wherein the smaller blows up the larger, is simply demonstrated by blowing soap bubbles on the ends of a $T$ tube. It occurred to me that the same might be demonstrated with toy rubber balloons attached to the ends of a $T$ tube. Accordingly, I fixed securely two balloons to the two arms of a glass $T$ tube, and the third arm I connected to a hand pump. With this arrangement I proceeded to pump.

Can the reader conjecture on the results of this endeavor? In my own case they were totally alien to my expectations!

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# ANNOUNCEMENTS AND NEWS

## Book Reviews

**Atomic Energy in War and Peace.** CAPTAIN BURR W. LEYSON. Pp. 217. E. P. Dutton and Company, Inc., New York, 1951. Price $3.75.

This is a volume which is avowedly popular in nature. It is designed to give the nontechnical reader some feeling for the tremendous developments that have taken place during the last ten years in the growth of the science of the nucleus and in the application of that science to practical, though often destructive, ends. For this very reason, it is unlikely to satisfy a trained physicist. Nevertheless, every good scientist who reads a popular book hopes to find in it a few sentences or a few phrases that illuminate a difficult problem in a manner that he has not thought of himself. One can think of the writings of Eddington and Jeans with the greatest respect, for at every page there is some apt illustration or some telling analogy that the physicist can store away in his mind to aid subsequent explanations to elementary students.

The present volume, however, lacks many of the features which would be necessary to make it a classic. It begins with the usual popular multicolored depiction of the atom. It gives amazingly detailed descriptions of the nuclear processes of radioactivity, and then divulges some of the unclassified secrets of the atomic bomb including its operation and effects. Following this is a brief account of the production of fissionable materials by gaseous diffusion and the chain-reacting pile. Another chapter contains a short pronouncement concerning the theory of and materials for the hydrogen bomb, and the following three chapters repeat and expand what was written in an earlier chapter concerning the effects of atomic bombs. A useless chapter on radiation detectors precedes an interesting popular account of particle accelerators. A chapter which claims to describe recent atomic energy research goes astray on other research and includes, as well, a short sermon on economics. The final chapter describes a few interesting applications of radioactive tracers in industry and medicine.

On the whole the book does not possess the clarity and accuracy which the book jacket claims. The eleven chapters of the book seem only loosely tied together. The individual chapters have no subheadings and, as a result, quite different topics often pass imperceptibly one into another.

The book is inclined to be repetitious and often wastes words: e.g., p. 84, "Having greater velocity it follows that they will travel faster—." The work includes no references or bibliography. The figures are neither numbered nor individually credited. There are also several misprints.

The book does not contain a preface and nowhere in the book is it made clear for whom the book is intended. In the opinion of the reviewer, the book was not written for physics teachers or physics students because the loose phraseology which is often employed will cause the former to cringe and the latter to be corrupted. For example, in the glossary (which strives to be a valuable addition to the book) is found "MASS: Term used to describe weight or volume." Also, the unfortunate phrase "degree of heat" is found again and again.

Although the title reads "Atomic Energy in War and Peace," by far the greater portion of the book deals with atomic energy in war. The merit of the book may lie in its rational account of the effects of atomic bombs and its description of several simple defense expedients.

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**How to Study, How to Solve.** H. M. DADOURIAN. Second Edition, Pp. 121+vi, Figs. 70. Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1951. Price $0.60.

This book is a small paper-covered volume which could easily slip into one's pocket. In it Mr. Dadourian tries to offer help to the student in three areas, and hence he divides the book into three parts. In the first part he discusses how to study. This discussion is fairly general, and the suggestions are applicable to the study of almost any subject. In the second part the author discusses how to solve mathematical problems and gives examples from courses ranging from elementary algebra to calculus. In the third part he reviews some of the basic concepts of the various mathematics courses from arithmetic to calculus.

There are many excellent suggestions to the student in this little book. None of them are particularly new or original. They are the same suggestions which experienced teachers are continually giving to their students. However, there probably is an advantage in having them written. An example of an often repeated but commonly ignored

caution is the author's final suggestion about problem solving: "Use your common sense at every step of the analysis. At each step ask yourself if the step you have just taken is sensible and then, at the end, whether the final result is reasonable."

The first two parts of the book would be useful to college students who want or need specific study suggestions and to inexperienced teachers who are anxious to help their students. I doubt if the book would be readable by high school students, particularly by those who can neither study nor solve effectively.

After the suggestions for studying and solving, this book devotes Part 3 to summaries and lists of important facts and formulas from arithmetic, algebra, trigonometry, analytic geometry, and calculus. These might be quite useful for someone who has mastered these courses and wishes a short review. The author has pointed out in this section the important key concepts in each course which would help the reader to recognize these apart from the maze of detail and mechanical operations he may have encountered as he studied these courses.

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**A Concise History of Astronomy.** PETER DOIG. Pp. xi+311. 13.5×20.5 cm. Philosophical Library, Inc., New York, 1951. Price $4.75.

The past several decades have witnessed a most unusual stride in the revolutionary advancement of the physical sciences. In this respect, astronomy indeed occupies a foremost position. This assumption seems reasonable when we consider the staggering expansion of man's growing universe through the last two centuries. Even an informed man, 200 years ago, was still immersed in the illusion of a world system not widely different from an idyl of the ancient Greek universe. Kepler himself still nourished the notion that the stars were part of a solid shell of the heavens. Isaac Newton, the wizard of classical astronomy, also hardly fathomed the crushing immensities of cosmic space which, with its millions of light years, was to invade the human horizon in the succeeding two centuries.

Little wonder then that astronomers had neither the time nor the inclination to look backward, nor even to become concerned in the pace of man's growing knowledge of the cosmic scheme. As fast as celestial science progressed, just so neglected and forlorn remained man's knowledge of its history. All textbooks of astronomy are the sad and alarming evidence that we leave history only to the military and political strategists. P. Duhem's French monumental history of astronomy, ending with Copernicus, came out in seven volumes, yet unfinished, in the first decade of the present century. Since Galileo, the growth of the science of astronomy obtained only fragmentary expositions in various minor works, usually reflecting a somewhat exclusive atmosphere, when written in French, German, or English.

After an absence for about half a century of any historical work covering astronomy through the ages, Peter Doig's *Concise History of Astronomy* is an excellent attempt at the orientational presentation of the panoramic view of man's growing knowledge and penetration of the universe from the dawn of civilization through the centuries. It is the first history of astronomy trying to give an up-to-date account of our twentieth century and the role and contribution of the great American observatories toward the foundation of modern astrophysics. Ancient astronomy is treated by Peter Doig with an appropriate sobriety. This is evidenced by occasional references to O. Neugebauer, a leading authority on Egyptian and Babylonian astronomies, which were at the cradle of the ancient Greek astronomy. However, there is no full justification for inserting in a concise history the sweeping statement that we owe very little to Hindu astronomy. This may be seriously misleading to the uninformed reader. The same could also be said about the Mohammedans and Tartars, for whom the author reserves a separate chapter. Such judgments are shaped by an ever-changing character of opinion.

The story of the emergence of classical and modern astronomy is built up in Doig's book around a few great pioneers, so that Copernicus, Galileo, Newton, and Herschel head special chapters. Remaining chapters are arranged chronologically, from the eighteenth through the twentieth centuries. This may be a feasible but not quite a systematic approach. In such a presentation the torrential exposition of the rise of various fields of research altogether endangers the clarity of general orientation. While the cooperative blending of all will give a good picture on the one hand, yet a chapter on such topics as spectroscopy or photometry of the stars would, in itself, present a fascinating story. To build a story about a few intellectual giants is an excellent idea, and becomes very helpful particularly when entering a period of beehive scientific activity as witnessed since the turn of this century. Yet, as we approach the contemporary period, selection becomes more involved and the task of writing a sufficiently objective history, particularly of a concise character, more difficult. P. Doig's work, in this respect, is an illuminating indication that to write a history of modern astronomy even in brevity is an immense task, which will be approximated by enduring attempts of a number of scientific historians out of an international family.

It is surprising to note in a history of astronomy—a science which has much to do with navigation—the confusion between two countries in Europe. On page 58 we read that Kepler "had to go to Hungary where he visited Tycho two years afterwards in Prague." Furthermore, it would be beneficial to both laymen and scientists if the astronomers included were more humanized, with hints here and there of their characters and ways of life. In this manner, the history of science might play a very important role in shedding more light of deeper understanding on the philosophy of life. Thus, it would be illuminating to contrast Laplace and Newton, for such a comparison contains many an answer to some deceptive or tyrannical dictates of dialectic materialism.

Under all circumstances, however, P. Doig's *Concise History of Astronomy* is a most welcome reference book. It should lend encouragement to enduring efforts in creating a history of astronomy, both ancient and modern, yet to be written.

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**The Growth of Physical Science.** SIR JAMES JEANS. Second Edition, Pp. 364+x, Figs. 52, 14×19.5 cm. Cambridge University Press, New York, 1951. Price $3.75.

The enthusiastic readers of Jeans' earlier books will not be disappointed in this last one. The subject of the growth of physical science was an obvious one for a scientific author of his literary skills, and the result is the authoritative and interesting book one would expect.

Jeans saw the first edition in galley proof before he died, but this second edition is published posthumously, edited and corrected by the careful hand of P. J. Grant of the Cavendish Laboratory.

In this book Jeans presents to the layman a survey of the historical development of astronomy, mathematics, and physics from the dawn of civilization to the modern era. To do this in the space of about 350 pages would obviously require a great deal of omission and simplification, but it provides the reader with a coherent and unified picture which makes both pleasant and instructive reading.

This reviewer would quarrel somewhat with the author's use of short biographical sketches of major and minor scientists as a device to show historical development. The resulting concentration on the details of men's lives breaks up the continuity of discussion of the physical concepts and tends to detract from the clearest understanding of the main subject. The interesting personal bits about individual scientists certainly add to the readability and charm of the exposition, but if biographical details were introduced as secondary to the science, instead of the reverse—as happens in several cases in this book—the purpose as announced by Jeans' title would be more forcibly served.

This kind of book, whose writing requires so much time spent in finding the necessary historical material, reveals the acute need in the field of the history of science for a modern and authoritative compilation of biographical material which is both easily accessible and historically accurate. Much of the source material generally available is copied over and over again, often from original work done in the era when biographers felt the necessity of putting their subjects in the most favorable light possible. People who carry out biographical research usually become experts on the lives of a very few individuals but are often made most unhappy by the continued copying and recopying of the same inaccuracies and/or significant omissions.

A real service could be rendered if a properly edited biographical encyclopedia of scientists were available, to which the individual items were contributed by competent experts on particular subjects rather than compiled by scholars in the general historical field who turn to the same old sources for their material. This is not an adverse criticism of the present book, as no such encyclopedia exists, but for those particular scientists with whose lives this reviewer is intimately familiar, he can easily recognize the source material which in some details could be much improved.

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**Magnetic Materials.** F. BRAILSFORD. Second Edition revised. Pp. 156+ix, 16.5×10.5 cm. Methuen and Company, Ltd., London; John Wiley and Sons, Inc., New York, 1951. Price $1.50.

This little book is intended to give the advanced student, the research worker, and those concerned with the technological applications of magnetic materials, a comprehensive outline of the present state of knowledge of the subject. In this it only partly suceeds, but in view of its small compass and high selectivity, this could hardly be avoided. It represents another of the series of books which have in recent years been emerging from industrial laboratories and, like others of the series, is best in those parts to which the author has himself contributed. In this case, the treatment is most complete and satisfactory in respect to "losses" in sheet material and in respect to permanent magnets. In the latter subject it presents the best summary and the most detailed information regarding modern materials that has been seen by the reviewer. The theoretical parts which are included are somewhat scattered and are of uneven quality. One trivial but confusing mistake is in the sign of the potential energy of a magnet deflected from parallelism with an applied magnetic field (p. 3). The discussion of domain boundary displacement by applied fields (pp. 43–47) is extremely neat and easy to understand.

There are numerous figures but these suffer from a fault which is almost inescapable in a book with such small pages. They are rather diagrams than quantitative presentations of data. A little more care with the figures would have improved the book considerably. One slip which may be caused in part by the aforementioned difficulty regarding figures is on page 8 wherein curves (*d*) for *B—H* and *B* appear to converge toward the *H* axis whereas, of course, the horizontal distance between them increases as they approach zero ordinate.

In the discussion of the effect of alloying upon the saturation magnetization of nickel, the anomalous behavior of magnanese is not explained as it might have been by the fact that manganese may also contribute "holes" in the 3*d* level (p. 116). It is rather surprising that magnetic shielding which is listed as one of the important technical applications of high permeability materials is not further discussed. Altogether, however, the book is interesting, well written, and reliable in technical detail. *Methuen's Monographs on Physical Subjects*, of which series this is a member, have a good reputation as cramming aids for passing examinations, and this specimen should be excellent for this purpose.

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**Radioactivity Applied to Chemistry.** A. C. WAHL AND N. A. BONNER. Pp. 604+xv, Figs. 58, Tables 64, 15×23.5 cm. John Wiley and Sons, Inc., New York, Chapman and Hall, Ltd., London, 1951. Price $7.50.

*Radioactivity Applied to Chemistry* is a well-planned book which adequately fulfils its stated purposes: to aquaint research workers with the various experimental fields of

chemistry in which radioactivity is utilized, and to present tables of data which would be useful in applying the fundamentals of radioactivity to research. The book is divided into two parts, the first of which consists of ten chapters concerned with the principles and ideas involved in the application of radioactivity to chemical research. The topics, each of which was covered by a competent authority, are: isotope exchange reactions, radioactivity applied to chemical kinetics, radioactivity applied to structural chemistry, radioactivity applied to self-diffusion studies, radioactivity applied to analytical chemistry, behavior of carrier-free tracers, radioactivity applied to the investigation of the newer elements, chemical phenomena accompanying nuclear reactions (hot-atom chemistry), emanation methods, and radioactivity applied to surface determinations.

The second portion of the book contains 178 pages of tables covering factual information needed to apply the principles discussed in the ten chapters. These tables have been set up both in number and title to supplement the material in the first part of the book. Data taken from over 1500 papers and books, appearing through 1949, are presented in the tables, and all of these references are catalogued as a bibliography which follows the tables. This book is thus an especially worthwhile reference volume for research workers in the fields of radioactivity applied to chemistry.

As a text the book appears to be of value only in an advanced course because of the rigorous, detailed treatment of material. However, many teachers of courses in chemistry or radiochemistry, whether beginning or advanced, would find portions useful as a source of material. For instance, in the chapter entitled "Radiochemistry Applied to the Discovery and Investigation of the Newer Elements" by C. S. Garner, an interesting historical development of the work leading to the isolation and naming of elements such as technetium, promethium, neptunium, berkelium, and others is presented. It would also appear that certain parts of the book would be valuable for analytical chemistry and biochemistry teachers. The chapter on "Radioactivity Applied to Analytical Chemistry" by V. J. Linnenbom indicates several instances in which radioisotopes may be applied in analytical procedures which could be widely useful and which will probably be considered standard methods in the near future. On the other hand, the first chapter, "Isotope Exchange Reactions" by O. E. Myers and R. J. Prestwood, while it contains many useful and interesting facts, is designed mainly for the advanced student or research worker. This book might also be used as a basis for topics in a seminar in that its subject matter is fundamental and the material covered is quite recent. It is also an advantage that the book is written in a clear and concise manner and few typographical errors are to be found.

This volume will be an excellent addition to the libraries of those interested in radioactivity and its application to chemical research.

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## Books Received

**Advanced Fluid Dynamics and Fluid Machinery.** R. C. BINER. 426 pp. Prentice-Hall, Inc., New York, 1951. $6.00.

**Aircraft Jet Powerplants.** FRANKLIN P. DURHAM. 326 pp. Prentice-Hall, Inc., New York, 1951. $5.00.

**Annual Review of Physical Chemistry, Volume II.** Edited by G. K. ROLLEFSON. 462 pp. Annual Reviews, Inc., Stanford, California, 1951. $6.00.

**Astrophysics.** Edited by J. A. HYNEK. 703 pp. McGraw-Hill Book Company, Inc., New York, 1951. $12.00.

**The Atomic-Molecular Theory, Case 4.** LEONARD K. NASH. 115 pp. Harvard University Press, Cambridge, Massachusetts, 1950. $1.35.

**Attitudes of Scientists and Engineers about Their Government Employment, Volume I.** 223 pp. The Maxwell Graduate School of Citizenship and Public Affairs for the Office of Naval Research, Syracuse University, Syracuse, New York, 1950.

**The Aurorae.** L. HARANG. 166 pp. John Wiley & Sons, Inc., New York, 1951. $4.50.

**Basic Electrical Measurements.** MELVILLE B. STOUT. 504 pp. Prentice-Hall, Inc., New York, 1950. $5.75.

**Borderlands of Science.** ALFRED STILL. 424 pp. Philosophical Library, Inc., New York, 1950. $3.75.

**Calculus and Analytic Geometry.** GEORGE B. THOMAS, JR. 685 pp. Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1951. $6.00.

**Careers in Physics.** ALPHEUS W. SMITH. 271 pp. Long's College Book Company, Columbus, Ohio, 1951. $4.00.

**Chambers's Dictionary of Scientists.** A. V. HOWARD. 499 pp. E. P. Dutton & Company, Inc., New York, 1951. $4.00.

**Chemical Engineering Catalog 1949–50. (34th Edition.)** 1580 pp. Reinhold Publishing Corporation, New York, 1949.

**Classical Mechanics.** H. C. CORBEN AND PHILIP STEHLE. 388 pp. John Wiley & Sons, Inc., New York, 1950. $6.50.

**The Classical Theory of Fields. (Second Edition.)** L. LANDAU AND E. LIFSCHITZ. (Translated from the Russian by M. Hamermesh.) 354 pp. Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1951. $7.50.

**Cloud Physics.** D. W. PERRIE. 119 pp. John Wiley & Sons, Inc., New York, 1950. $4.50.

**Cours de Physique.** J. SIMON AND G. POUX. 400 pp. J. B. Bailliere & Fils, Paris, 1951.

**Creep of Metals.** L. A. ROTHERHAM. 80 pp. Institute of Physics, London, 1951. 15s.

**Defense Programs of Selected Federal Agencies Affecting Colleges and Universities.** MARGARET C. JAMES, WILLIAM J. MCGLOTHLIN, AND HARRY B. WILLIAMS. 56 pp. Board of Control for Southern Regional Education, Atlanta, Georgia; 1951. $.40.

**The Earth's Magnetism. (Second Edition.)** SYDNEY CHAPMAN. 127 pp. John Wiley & Sonc, Inc., New York, 1951. $1.50.

**Electricité. (Fourth Edition.)** Y. ROCARD. 538 pp. Masson et Cie, Editeurs, Saint-Germain, Paris, 1951. 2200 Fr.

**Electromagnetic Problems of Microwave Theory.** H. MOTZ. 184 pp. John Wiley & Sons, Inc., New York, 1951. $2.00.

**Elementary Problems in Engineering.** H. W. LEACH AND GEORGE C. BEAKLEY. 269 pp. The Macmillan Company, New York, 1951. $3.50.

**The Elements of Thermodynamics.** CHARLES FABRY. (Translated by G. F. Herrenden Harker from 6th French Edition.) 229 pp. Frederick Muller, Ltd., London, 1951. 15s.

**Essentials in Problem Solving.** ZUCE KOGAN. 79 pp. Zuce Kogan, Chicago, Illinois, 1951. $3.00.

**Ferro-Magnetism.** RICHARD M. BOZORTH. 968 pp. D. Van Nostrand Company, Inc., New York, 1951. $17.50.

**Fourier Transforms.** IAN N. SNEDDON. 542 pp. McGraw-Hill Book Company, Inc., New York, 1951. $10.00.

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**Fundamental Mechanisms of Photographic Sensitivity.** Edited by J. W. MITCHELL. 347 pp. Butterworths Scientific Publications, London, 1951. $9.50.

**Fundamentals of Electronics.** F. H. MITCHELL. 243 pp. Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1951. $4.50.

**General Education Board Annual Report 1950.** 91 pp. General Education Board, New York.

**Heat and Thermodynamics. (Third Edition.)** MARK ZEMANSKY. 465 pp. McGraw-Hill Book Company, Inc., New York, 1951. $6.00.

**The Hebrew Impact on Western Civilization.** Edited by DAGOBERT D. RUNES. 922 pp. Philosophical Library, Inc., New York, 1951. $10.00.

**High Frequency Transmission Lines. (Third Edition.)** WILLIS JACKSON. 152 pp. John Wiley & Sons, Inc., New York, 1951. $1.75.

**History of the Theories of Aether and Electricity, Volume I. (Second Edition Revised.)** SIR EDMUND WHITTAKER. 434 pp. Thomas Nelson and Sons, Ltd., New York, 1951. 32s 6d.

**The Identification of Molecular Spectra. (Second Edition Revised.)** R. W. B. PEARSE AND A. G. GAYDON. 276 pp. John Wiley & Sons, Inc., New York, 1950. $8.50.

**Improving College Instruction.** Edited by FRED J. KELLY. 195 pp. American Council on Education, Washington, D. C., 1951. $1.75.

**Initiation and Growth of Explosion in Liquids and Solids.** F. P. BOWDEN AND A. D. YOFFE. 102 pp. Cambridge University Press, New York, 1951.

**Integral Transforms in Mathematical Physics.** C. J. TRANTER. 118 pp. John Wiley & Sons, Inc., New York, 1951. $1.50.

**An International Bibliography on Atomic Energy, Volume II.** ATOMIC ENERGY COMMISSION GROUP. 433 pp. Columbia University Press, New York, 1950. $10.00.

**An Introduction to Acoustics.** ROBERT H. RANDALL. 340 pp. Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1951. $6.00.

**Introduction to Number Theory.** TRYGVE NAGELL. 309 pp. John Wiley & Sons, Inc., New York, 1951. $5.00.

**An Introduction to Electron Optics.** L. JACOB. 150 pp. John Wiley & Sons, Inc., New York, 1951. $2.00.

**An Introduction to the Science of Photography.** KATHERINE CHAMBERLAIN. 291 pp. The Macmillan Company, New York, 1951. $4.75.

**Introduction to the Study of Aircraft Vibration and Flutter.** ROBERT H. SCANLAN AND ROBERT ROSENBAUM. 428 pp. The Macmillan Company, New York, 1951. $7.50.

**Isoperimetric Inequalities in Mathematical Physics, Study 27.** G. POLYA AND G. SZEGO. 279 pp. Princeton University Press, Princeton, New Jersey, 1951. $3.00.

**Johannes Kepler: Life and Letters.** CAROLA BAUMGARDT. 209 pp. Philosophical Library, Inc., New York, 1951. $3.75.

**Laboratory Experiments in the Physical Sciences.** ANTON POSTL. 72 pp. Burgess Publishing Company, Minneapolis, Minnesota, 1941. $1.50.

**Linear Computations.** PAUL S. DWYER. 344 pp. John Wiley & Sons, Inc., New York, 1951. $6.50.

**Mathematical Engineering Analysis.** RUFUS OLDENBURGER. 426 pp. The Macmillan Company, New York, 1950. $6.00.

**Mathematics Queen and Servant of Science.** ERIC TEMPLE BELL. 437 pp. McGraw-Hill Book Company, Inc., New York, 1951. $5.00.

**The Natural Sciences in Western Germany.** KURT UEBERREITER. 127 pp. Library of Congress, European Affairs Division.

**A New Theory of Gravitation.** DR. JAKOB MANDELKER. 25 pp. Philosophical Library, Inc., New York, 1951. $2.75.

**Panorama of Science.** Edited by WEBSTER PRENTISS TRUE. 416 pp. The Series Publishers, Inc., New York, 1951.

**Phase Microscopy, Principles and Applications.** ALVA H. BENNETT, HAROLD OSTERBERG, HELEN JUPNIK, AND OSCAR W. RICHARDS. 320 pp. John Wiley Sons, Inc., New York, 1951. $7.50.

**Phase Transformations in Solids.** Edited by R. SMOLUCHOWSKI, J. E. MAYER, AND W. A. WEYL. 660 pp. John Wiley & Sons, Inc., New York, 1951. $9.50.

**The Photographic Study of Rapid Events.** W. D. CHESTERMAN. 200 pp. Oxford University Press, London, 1951. $4.25.

**The Physical Sciences. (Third Edition.)** EMMETT JAMES CABLE, ROBERT WARD GETCHELL, WILLIAM HENRY KADESCH, AND HARRY E. CRULL. 496 pp. Prentice-Hall, Inc., New York, 1951. $7.35.

**Physics in Chemical Industry.** R. C. L. BOSWORTH. 928 pp. The Macmillan Company, New York, 1951. $12.75.

**Physics of Lubrication.** 96 pp. The Institute of Physics, London, 1951. 15s.

**Quantum Mechanics.** ALFRED LANDE. 307 pp. Pitman Publishing Corporation, New York, 1951. $5.50.

**Quantum Theory.** DAVID BOHM. 646 pp. Prentice-Hall, Inc., New York, 1951. $7.50.

**Radiation Monitoring in Atomic Defense.** DWIGHT E. GRAY AND JOHN H. MARTENS. 122 pp. D. Van Nostrand Company, Inc., New York, 1951. $2.00.

**Real Academia de Ciencias Exactas.** ALFREDO PÁEZ BALACA, OBDULIO FERNÁNDEZ AND M. GARCIE DE MIRASIERRA, ANDRÉS LEÓN MAROTO, ANGEL MALLOL GARCIA. 114 pp. Domicilio de la Academia, Madrid, 1950.

**Research Corporation.** Annual Report for 1950. 63 pp.

**Scientific Russian.** JAMES W. PERRY. 816 pp. Interscience Publishers, Inc., New York, 1951. $7.50.

**Simplified Mechanics and Strength of Materials.** HARRY PARKER. 275 pp. John Wiley & Sons, Inc., New York, 1951. $4.00.

**Space—Time—Matter.** First American Printing of the Fourth Edition. HERMAN WEYL. (Translated by Henry L. Brose.) 330 pp. Dover Publications, Inc., New York, 1951. $3.95.

**Space-Time Structure.** ERWIN SCHRODINGER. 119 pp. Cambridge University Press, New York, 1950. $2.75.

**Static and Dynamic Electricity. (Second Edition.)** WILLIAM RALPH SMYTHE. 616 pp. McGraw-Hill Book Company, Inc., New York, 1950. $8.50.

**Statistical Methodology Reviews, 1941–1950.** Edited by OSCAR KRISEN BUROS. 457 pp. John Wiley & Sons, Inc., New York, 1950. $7.00.

**Superfluids.** FRITZ LONDON. 161 pp. John Wiley & Sons, Inc., New York, 1950. $5.00.

**Technical Publications 1948.** Edited by ARMAN E. BECKER. 512 pp. Standard Oil Company (New Jersey) and Affiliated Companies, 1951.

**Theologie Und Physik.** HEIMO DOLCH. 110 pp. Verlag Herder, Freiburg, Germany, 1951.

**Theory of Perfectly Plastic Solids.** WILLIAM PRAGER AND PHILIP G. HODGE, JR. 264 pp. John Wiley & Sons, Inc., New York, 1951. $5.50.

**Thermodynamics. (Second Edition.)** GEORGE A. HAWKINS. 563 pp. John Wiley & Sons, Inc., New York, 1951. $6.50.

**Thermodynamics. (Fourth Edition.)** A. W. PORTER. 124 pp. John Wiley & Sons, Inc., New York, 1951. $1.50.

**Time's Arrow and Evolution.** HAROLD F. BLUM. 222 pp. Princeton University Press, Princeton, New Jersey, 1951. $4.00.

**200 Miles Up.** J. GORDON VAETH. 207 pp. Ronald Press Company, New York, 1951. $4.50.

**Universal Mechanics and Hamiltons Quaternions.** OTTO F. FISCHER. 356 pp. Axion Institute, Stockholm, 1951. 52kr.

**University of Utah Centennial Commemoration Proceedings.** 133 pp. University of Utah Press, Salt Lake City, 1950.

**The Young Scientist.** MAITLAND P. SIMMONS. 164 pp. Exposition Press, New York, 1951. $3.00.

## New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. **20**, 4 (1952)].

**Allen,** Louise Jane (*J*), Nuclear Research Bldg., McMaster University, Hamilton, Ontario, Canada

**Boley,** Forrest Irving, Scott Laboratory, Middletown, Conn.

**Brown,** Frederick Calvin, Reed College, Portland 2, Ore.

**Buynak,** George Richard, 1558 Elmwood Avenue, Buffalo, N. Y.

**Carpenter,** Delma Rae, Jr., Box 704 Virginia Military Institute, Lexington, Va.

**Cohen,** Robert S., Scott Laboratory, Wesleyan University, Middletown, Conn.

**Deeley,** Charles William (*J*), 143 Wesleyan Station, Middletown, Conn.

**Gault,** Robert Louis (*J*), 903-D Cherry Lane, East Lansing, Mich.

**Koranda,** Edward, 33–38 Chowan Avenue, Rt. 3, Lincolnia Park, Alexandria, Va.

**Price,** Robert Louis, 403 Second Avenue, Joliet, Ill.

**Rowe,** William David (*J*), Sigma Chi House, Wesleyan University, Middletown, Conn.

**Sandin,** Eric Verner, Jr. (*J*), Scott Laboratory, Wesleyan University, Middletown, Conn.

**Stone,** Paul W., 2809 Norbert Street, Flint, Mich.

**Taylor,** Julius Henry, Morgan State College, Baltimore 12, Md.

# RECENT MEETINGS

## Chesapeake Section

On October 4, 1951, the Chesapeake Section met at the Roosevelt High School in Washington, D. C., and a panel discussion of interest to secondary schools and university teachers of physics was held. The discussion was stimulated by the following two questions:

1. Can the university physics staffs help the secondary school physics staffs become more effective?

2. Can the secondary school physics staffs help the university physics staffs become more effective?

The speakers comprising the panel are listed below:

R. H. JOHNSON, *Baltimore City College.*
R. MORGAN, Chairman, *University of Maryland.*

R. E. LATHAM, *Episcopal High School, Alexandria, Virginia.*
G. M. KOEHL, Asst. Dean, *George Washington University.*
RALPH WATT, *Eastern High School, Washington, D. C.*
R. R. MEIJER, *George Washington University.*

There was an active participation by the audience during the discussion. It was apparent that not all the teachers at the secondary and college level have the same concept of how physics should be taught. In spite of the variety of opinions, a number of valuable suggestions were made relative to the assistance that might be given to secondary teachers by staff members at the college level. It was generally agreed by all that the mathematical skills possessed by students at both the secondary and college level were inadequate.

J. H. McMILLEN, *Secretary*